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Mathematical Principles,

CONTAINING THE

ELEMENTS OF GEOMETRY.

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Mathematical Principles,

CONTAINING THE

ELEMENTS OF GEOMETRY,

SO DIGESTED

As to render both the practical and theoretical Parts familiar and eaſy to Youth;

AND,

To convey ſo much of the Science in general, as may be neceſſary to thoſe of riper Years, who may be reſtricted in point of Time.

THE WHOLE

Selected from EUCLID, and the moſt eminent Authors, and particularly calculated to be extremely uſeful in Schools.

By SISSON PUTLAND DARLING,
Principal of the Mercantile, Nautical and Military Academy, in Mabbot-ſtreet, DUBLIN.

Science may be ornamented, but not burthened with dreſs.
Encyclopædia.

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PREFACE.

TO add to the number of the Treatiſes already publiſhed on this ſubject, many of which are excellent, may, perhaps, be conſidered as oſtentatious and unneceſſary; wherefore it is in ſome degree requiſite to ſay a few words, as to the neceſſity of the following work, and in explanation of the motives which induced the Author to preſent it to the Public.

Long experience has ſufficiently convinced him, that ſomething might be offered as an introduction to Mathematics, more happily adapted to the convenience and capacity of learners, than has hitherto appeared; particularly to thoſe whoſe time and condition forbid them to enter at large into the ſcience: and, as that work which is comprehended with the leaſt difficulty muſt be moſt advantageous to perſons *juſt entering* upon any particular branch of learning, the Author is perſuaded this Treatiſe, in which the principles of Geometry are laid down in a manner new and familiar, and from experience, found to be intelligible even to weak capacities, muſt obtain a reception as favourable as he truſts its *deſign is* uſeful.

To convey to young and tender minds the principles of a ſcience, which depends not

only on the moſt critical demonſtration, but requires at leaſt moderate abilities to underſtand, muſt be a taſk of ſome difficulty and application; to put, therefore, into the hands of youth, or even of more adult, but inexperienced perſons, ſome Treatiſes on this ſubject already publiſhed, which, however learned and ingenious, are calculated *only* for perſons of like learning and ingenuity with their authors, muſt tend to retard the progreſs of thoſe whoſe advancement we wiſh to accellerate.

To remove this difficulty, the Author has particularly applied himſelf; and, he flatters himſelf, it will be found he has treated the ſubject with that familiarity of expreſſion, and perſpicuity of demonſtration, which will enable the young beginner to advance more ſpeedily towards knowledge, than he poſſibly could do by the mode of inſtruction neceſſarily adopted from the ſyſtems laid down in the treatiſes generally uſed; wherefore, this work may he found equally uſeful to the Teacher and the Pupil.

It is, however, by no means the Author's intention to depreciate the valuable publications of former writers, from whom he acknowledges to have ſelected many uſeful materials in the compilation of this Treatiſe; *Euclid* in particular, whom he had principally in view, and the many editions of his work by *Barrow*, *Simpſon*, *Cunn*, *Whiſton*, &c. which will be held in eſteem while

while Mathematics ſhall be thought worthy our attention and regard. What is now offered, is calculated only for the purpoſe already mentioned, *an introduction to the more extenſive writers;* yet, made to exhibit ſufficient inſtruction to thoſe who have not time or opportunity to peruſe them: for, to begin the ſtudy of the ſcience in ſuch, the Author, in many inſtances, has experienced to be exceedingly toilſome, and almoſt ſufficient to diſcourage a learner from further purſuits.

In a work, therefore, of this nature, the learned reader is not to expect, that matters ſhall be treated in a manner only ſuitable to his ſuperior capacity, the whole being adapted chiefly for the information of young and tender minds, the ſtyle is conſequently familiar, and the manner eaſy.

An experienced reader, already well acquainted with the ſubject, might, perhaps, object to the definition given to ſimilar figures. Thus we ſay, "*ſimilar* or *proportional* figures are ſuch as have all the angles in one figure equal reſpectively to thoſe of the other, and *that*, whether the ſides be equal or not." Such a reader might ſay, it ſhould alſo be mentioned, "that the ſides about the equal angles were proportional;" in this the Author agrees, and acknowledges the definition would, by the addition of ſuch words, be more compleat: but, when it is conſidered that the perſons for whoſe uſe this definition was chiefly intended, underſtand nothing of proportion, the

the inſertion of theſe words would render that obſcure, which, without them, has been found intelligible even to a child. The like may be obſerved reſpecting the definition of a *line*, *ſurface*, &c. which are here conſidered, not as magnitudes in themſelves, but, as ſeparate properties of *magnitude;* which term, it is hoped, has not improperly been applied to a ſolid only, and, without very great offence to an underſtanding geometrician, may be ſo conſidered, becauſe, having theſe three properties, *viz. length*, *breadth*, and *thickneſs;* of which the Author has conſidered the line as the firſt, the ſurface the ſecond, and the third inherent in the ſolid itſelf. At times, when he defined each as a magnitude in itſelf, as indeed each is generally ſo eſteemed, he found young minds confuſed with the definition, and therefore to make it intelligible to ſuch, was neceſſitated to give up a little, whereby the definition, as it now ſtands, became perfectly intelligible. However, theſe matters are obviated in the progreſs of the work; for the young pupil, as his underſtanding improves, will of himſelf perceive, and become able to ſupply any deficiency that may ariſe from the abovementioned circumſtances; *and, as a further apology to a learned reader, the Author begs leave to refer him to the obſervation on def.* 10, *page* 5.

With reſpect to the plan of the work itſelf, it is divided into three parts; in the firſt, every term in the ſcience, neceſſary for

for the knowledge of it, is explained, the definition of each term is given independently, and where it appeared neceſſary, an explanatory note is added, in order to fix it more permanently on the mind. This method of ſeparating the definition from its explanation, the Author has experimentally found to be exceedingly neceſſary, having had many capacities to deal with not able to underſtand the definition in itſelf, however ſimple: to clog, therefore, the definition with its explanation, made it impoſſible for ſuch an underſtanding to ſeparate the one from the other; for which reaſon it became neceſſary to give the explanation ſeparately, and he hopes he will ſtand excuſed, if in theſe explanatory obſervations he has, for the convenience of his uninformed readers, whoſe intereſt he has had chiefly at heart, enlarged ſomewhat more than to others might ſeem neceſſary, eſpecially when it is conſidered, that he who is ignorant of the terms, muſt remain ignorant of the art itſelf. And thoſe definitions being introductory to, and, as it were, the ground-work of the whole ſuperſtructure, without rendering them perfectly intelligible, familiar and eaſy, all further attempts towards improvement would be not only laborious, but ineffectual.

In the ſecond part is contained the practical problems; and here the Author has, contrary to the mode uſually purſued, purpoſely omitted the demonſtrations of each problem.

problem. Theſe he generally made evening exerciſes for his pupils, who daily produced a problem neatly drawn, and at the foot of it a demonſtration, the reſult of their own knowledge of the theoretical or third part; and, for their aſſiſtance in this particular, reference is made at the foot of each problem to the theorem, &c. whereon the proof depends. The pupil, by being thus able to draw the proof himſelf, becomes grounded in the knowledge of the ſcience *more certainly* than he could be, by reading the beſt proof that could be laid before him by another. But here care muſt be taken by the teacher, that the arguments he uſes be properly founded, as the young pupil may poſſibly err in a few problems at the beginning.

In this part, if time will not permit the pupil to go through every practical problem, the whole not being abſolutely neceſſary at his entrance upon any mathematical branch, the teacher may, in that caſe, eaſily ſelect ſuch only as are neceſſary for his immediate uſe, and leave the remainder to be done in the evenings, as above recommended, whereby the whole problematical part will be perfectly underſtood. The like may be obſerved with reſpect to the definitions, they being very numerous, before the ſecond part is entered upon; or, if the pupil ſhould be very much ſtinted in time, he may begin the problematical part at once, and, as the geometrical terms appear in each problem, he may write down

down its definition from the firſt part, and in that manner ſtudy each as it occurs. But it muſt be obſerved, that before he can enter upon the third or theoretical part, it is abſolutely neceſſary that a few practical problems of the ſecond part ſhould be known, *viz.* the 1ſt and 2d, the 4th to the 10th, the 12th, 16th, and 20th; and, as ſoon as he is capable of underſtanding the firſt 16 theorems, which include all the uſeful propoſitions in Euclid's firſt book, he may begin, but not before, to demonſtrate the practical part; and then he muſt undertake *only ſuch problems* as, in their proof, ſhall depend upon thoſe theorems, omitting the others, until he becomes more advanced in the theory. In this manner, under a proper tutor, the knowledge of Geometry will become eaſy and entertaining; the pupil, from thus purſuing the ſcience, will be able to apply his knowledge of the theory to practice; and ſhould new matter be ſtarted to him at any time, he will, from his knowledge of the theory, &c. be able to trace its principles, and inſtantly perceive the truth or falſity of it.

In the third part is contained many uſeful theorems ſelected from various writers, *viz.* *Euclid*, *Robertſon*, *Harris*, *Dogherty*, &c. &c. but from *Euclid* in particular, theſe follow nearly in the ſame order in which they are found in him, every where omitting the practical problems, becauſe they are arranged in the ſecond part. Beſides, the number of each problem, theorem, &c. there

there is placed a reference to the number of the problem, book, &c. in Euclid, whereby the reader may refer thereto, if his curioſity ſhould lead him ſo to do. In the arrangement of theſe theorems, the Author has been particularly attentive to the ſecond book of Euclid, which he has demonſtrated after Tacquet's method, *on the line only*, and annexed to each theorem a demonſtration in general terms; and alſo, every uſeful corollary with its demonſtration. He has been thus particular in this part of the work, for the advantage and convenience of the Students of Trinity College, many of whom were his pupils both before and ſince their entrance. Particular care has been taken, as to the number and figure of each theorem, which are the ſame as in *Whiſton*, whoſe edition of Euclid the Author has prefered, as being the one chiefly uſed in the College. The great reſpect he entertains for the members of that ſeminary, and his gratitude for the many inſtances of friendſhip he has experienced from ſome of the moſt diſtinguiſhed in the number, made him anxious to render them every ſervice in his power: if he has, in any degree, contributed to their advantage in this work, he will think his labour amply rewarded.

To conclude. His endeavours have been to promote inſtruction, by making it eaſy and agreeable, and to render the road to mathematical learning more ſhort and pleaſing than it has been generally found.

SYNOPSIS

OF THE

Several Propoſitions of EUCLID contained in the following Work; whereby, the *Reader* may have Reference as he may think neceſſary.

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The following Subſcribers Names came too late to be inſerted in their proper Places.

Thomas Wildridge,
John Peter Kemble,
Charles Tottenham, Eſq.
William Semple, 2 books,
John Fitzakerly.

The Reader is requeſted to alter with his Pen, the following ERRATA, before he begins.

Page	Line	
9	5	For, FBGD is the ſupplement of AF, read AF *is the ſupplement of* FBGD.
9	10	For, GAGB or BFDEG, read G*D*GB *or* BF*A*EG.
11	21	For, f b c d, read *f d b c*.
19	24	For, either acute, or obtuſe angled, read *either* right, *acute or obtuſe*, &c.
20	19	For, quadrilateral figure, read *parallelogram*.
21	7	For, perpendicularly and parallel, read *perpendicularly* to CD *and parallel*, &c.
22	28	For, equal parts, read *two* equal parts.
25	4	After increaſed, inſert *angles*.
26	6	For, but on the oppoſite ſides, read but *at* oppoſite parallels, and *on the oppoſite*, &c.
34	5	For, with another; it, read *with* another *of like denomination*, &c.
48	7 & 20	For, and any quantities between which it is placed, read *and when placed between any quantities*.
51	1	For, one ſide are ſaid to be equal, read *one ſide of the ſign of equality* (=) *are ſaid to be equal*.
57	9	For, def. 13, read *def.* 15.
	31	For, theo. 38*, read *theo.* 39*.
65	20	For, def. 3, read *def.* 13.
67	25 & 26	For, which equal, read *which make equal*.
68	27	For, AD, read *AB*.
75	4	For, draw the lines, read *through the points B, C*, and *G, draw the lines*.
76	1	For, ABCD, read *ACBD*.
77	14	For, EF, read *DF*.
84	2	For, D, read *d*.
90	3 & 4	For, a b and a d, read *B b and B d*,
	8	For, E b d, read *e B d*.
96	8	Dele (*cor.* 42, *Euc. lib.* 1.)

* In the courſe of printing the work, the 23d theo. page 160, was inſerted, which has occaſioned an error ſimilar to the above in the references to the theorems from the 23d upwards, from page 56 to 160; but, from page 160 to the end, all references are right.

ERRATA.

Page	Line	
100	1	For C H F, read *G H F.*
101	14	For, (per. theo. 38,) read *(per cor. 2, theo.* 39.)
108	1	Insert the words (14 *Euc. lib.* 2.)
109	16	For, c, read *C.*
122	26 & 27	For, more than, viz. the other proportions, read *more than the other proportions,* viz.
125	2	For, (fig. 68) read (fig. 67, *No.* 2.)
126	18	For, b F c, read *d F c.*
127	24	For, a b, read *d b.*
131	28	For, included ſide A C, read *included ſide A B.*
133	7	For, on both circles, read *in both circles.*
134	12	For, ∠ A = ∠ C, read ∠ *A* = ∠ *B.*
135	8	For, ∠ A C D, read ∠ *A C B.*
136		Theo. 8, the reference to the fig. is wanting; obſerve the like at theo. 9, and ſome others; but, as the number of each theo. prob, &c. and number of the figure in the plate is always the ſame, the error is therefore not ſo material.
137	3 & 4	For, d and e *are* on the ſame ſide, &c. read d *and* e *on the ſame ſide, &c.*
147	29	For, nine, read *ten.*
	30	For, ſix, read *ſeven.*
149	19	For, A I, read *B I.*
151	laſt line,	For, □ of the *other* part C B, read □ *of the part C B.*
152	9	For, (per theo. 7, and 17 Euc. lib. 2.) read *(per theo.* 17, *and* 1 *Euc. lib.* 2.)
160	9	For, equally, read *unequally.*
177	20 & 21	For, under *the whole* ſecant, and part, &c. read *under* one *ſecant* line *and part.*
182	25	For, 16 Euc. lib. 6, read 1 *Euc. lib.* 6.
184	32	For, △ A D E : △ D E B :: △ *A B E* : △ E D C, read △ *A D E* : △ *D E B* :: △ *A D E* : △ *E D C.*
187	27	For, ∠'d, read ∟*'d.*
192	28	For, within the circle, (by, &c. read *within the circle,* or *(by, &c.*

Mathematical Principles.

PART I.

GEOMETRICAL DEFINITIONS.

1. A *DEFINITION*, is a clear deſcription of any thing from its particular parts, ſo, as to fix an idea or image thereof on the mind, leaving it in no doubt of its form or properties.

2. *Geometry*, is that ſcience which conſiders the properties of magnitude, that is, *of any body, capable of being increaſed or diminiſhed.*

Geometry is both *ſpeculative* and *practical;* the former, conſiders the order and mutual dependance of the properties of magnitude; and the latter, applies them to the uſeful purpoſes of life. *In magnitude or continued quantity,* are to be conſidered three dimenſions, viz. *length, breadth,* and *thickneſs:* wherefore, by geometry is taught the properties of *lines, angles, ſuperficies* and *ſolids.*

3. *A point*, is the beginning, or firſt principle of magnitude; it is without parts, conſequently indiviſable,

as A. fig. 1.

A point is ſo infinitely ſmall, as not to be divided even in thought. However impoſſible ſuch an idea may appear to any one, yet, ſuch a notion is very neceſſary in geometry, for without it there is no proceeding: and although there is not in nature ſuch a thing as a *mathematical* point, yet, there exiſt things capable of extenſion, which may be treated as if not made up of parts, therefore uſed only as ſimple marks without any magnitude. Thus a ſtar, which we perceive at night, may be conſidered as a point of this kind; for, although it may have parts, yet they being ſuch as we cannot ſeparate, the imagination may therefore change this into a reality which does not exiſt; and, indeed, Geometricians have a right to adopt ſuch notions, as they ſerve to make their ſpeculations more intelligible.

This point, in practical Geometry, is repreſented by the point of a fine pencil, drawing-pen, or point of your compaſs, &c.

OF LINES.

4. *A line*, is the firſt property of magnitude; it has one dimenſion only, viz. *length*,

as A B. fig. 2.

A line may be conceived to be produced from the flowing or motion of a point in any direction; which point, becauſe it is without dimenſions, cannot produce any, except that which ariſes from its motion,

tion, viz. *length*; and, becauſe this motion may vary: hence there ariſe different ſpecies of lines, as

5. *A right line*, is the *ſhorteſt* that can be drawn between two points, which points always terminate the line, as A B. fig. 2.

That is, when the flowing, or moving point, moves in a ſtraight, regular, and unvariable direction; and, if your eye be conceived to be in the continuation of that direction, the two terminating points would cover or hide from your eye the whole line. And obſerve, if one right line be laid upon another, and, at the ſame time, the extreme points of both lines fall upon, and agree with each other, thoſe lines, from their thus agreeing, are ſaid to be *equal* to each other.

6. *A curved line*, is *not* the ſhorteſt that can be drawn between two points, as B C, or D E. fig. 3.

That is, when the flowing point moves in a varied direction; and in that caſe, let the eye be placed in what ſituation you pleaſe, the terminating points will not prevent your ſeeing ſome part of the line.

A line is always expreſſed by two letters, one at each of its extreme points, as AB, (fig. 2.); and lines are always eſtimated in inches, feet, yards, &c. that is, a line may be 16 inches, 50 feet, &c. long. There are ſeveral denominations of lines, each of which ſhall be defined in their proper places.

OF SURFACES.

7. *A ſurface* or *ſuperficies*, the ſecond property of magnitude, is the outſide

or term only, but no part of the body, or ſolid: it has two dimenſions, viz. *length* and *breadth;* it is bounded by lines,

as A B C D. fig. 4.

A ſurface may be conceived to be produced from the flowing, or motion of a *line;* and as a line is without *breadth*, it cannot therefore produce *thickneſs*, conſequently it is no part of a ſolid, but only its term or boundary. From its length, ariſes the breadth of the ſurface, and from its motion (contrary to the continuation of itſelf) the length. And, becauſe this motion of the line may vary; hence ariſes different ſurfaces, as

8. *A plane ſurface*, or *a plane*, is that which lies evenly between its extreme lines; and, if a right line or the edge of a ſtraight rule be laid any where upon it, it will *every where* touch it,

as A B C D. fig. 4.

That is, a plane is a regular and even ſurface, produced from the flowing or motion of a right line, as A C (fig. 4) in a ſtraight and unvaried direction along the line C D; and, becauſe it is thus formed, the terms, or bounds of it, muſt be lines. Again, if your eye be conceived to be in the continuation of that direction, that is, of the line C D, in that caſe, the deſcribing line A C, would cover, or hide from your view *every part* of the plane, and repreſent to your eye only one line; and this motion would only appear by its approach to, or receſs from, your eye; that is, during its *approach* it would appear to *lengthen*, and in its *receſs* to grow *ſhorter*.

9. *A*

9. *A curved ſurface*, is that which does *not* lie evenly between its extreme lines, conſequently it differs in every reſpect from the plane ſurface,

as A or B. fig. 5.

For, it is produced from the flowing line moving in a variable direction. In a *curved* ſurface, the flowing line, may be either a curved, or a right line; as the ſurface A or B may be produced from the motion of the right line *a b*, along the curved line *a c*; or, by the curved line *a c*, along the right line *a b*. Again, the curved ſurface A B C D (fig. 7) is formed from the motion of the curved line B A D, or B C D, round its extreme points B, and D, as fixed.

10. *An irregular ſurface*, is that which hath no *uniformity* in it, but is compounded of the *plane* and the *curve* together,

as C. fig. 5.

Regular ſurfaces, are expreſſed by letters placed one at each corner, as A B C D;* (fig. 4) *or, for brevity, by two letters at the oppoſite corners, as A D or C B; either of which latter expreſſions will be more convenient than the other.*

* The word *angle* would here, perhaps, be more proper; but, as an angle is not yet *defined*, it would not be ſo plain to ſome readers; nor ſhall I, as I proceed, make uſe of any term or word I can avoid, until it is firſt *explained* or introduced in its order.

OF SOLIDS.

11. *A ſolid* or *magnitude*, is that which hath three dimenſions, viz. *length*, *breadth* and *thickneſs*.
as ABCDEFGH, or ABCD. fig. 6, & 7.

Solids in general may be conceived to be produced from the motion of a ſurface in any direction; they are therefore bounded by one or more ſurfaces, and are variouſly denominated, according to the form and motion of the ſurface which produces it: thus, the ſolid ABCDEFGH (fig. 6) is produced from the motion of the ſurface DE, along the ſurface GE, in an upright poſition, and from this parallel and even motion, it is called a *parallelopipedon*; and the ſolid ABCD (fig. 7) being produced from the motion of the plane ſurface ABC round itſelf upon the line AC, its *axis*, is called a *globe* or *ſphere*, &c.

The various ſolids produced from the motion of different ſurfaces are too numerous to mention here; but will be particularly treated of before Spheric Trigonometry *, *which will make a ſecond part to this work.*

OF FIGURES AND THEIR LINES.

12. *A figure*, is a bounded, or limited ſpace.

* The author, according to the encouragement he receives for this publication, means to publiſh two volumes more, one on plain and ſpherical trigonometry, the other on navigation and fortification, &c.

A figure

A figure is generally expreſſed by one letter placed within it, as A, B, C, D, &c. (fig. 8.)

13. *A plane figure*, is a plane ſurface bounded by one or more lines,
as A, B, C, D, E, F, G. fig. 8.

14. *A circle*, is a plane ſurface, bounded by one *uniformly* curved line, called its *circumference*, which is every where equally diſtant from a point (C) within it, called its *center*,
as A B C D E. fig. 9.

The circle may be conceived to be produced from the motion of the right line C D (fig. 9) on a plane, round one of its extreme points fixed, as C, which point, from its being fixed, gives the center; and the other extreme point D, during its motion, forms the circumference. From this formation of a circle it will not be difficult to underſtand, that all lines drawn from the point C, (or the center,) to its circumference, will be *equal*; for it is clear, they muſt all be equal to C D, the line with which the circle was deſcribed.

15. *The radius*, of a circle, is a right line, drawn from the center of the circle to its circumference,
as C A, C B, or C D. fig. 10.

As the center of a circle is every where equally diſtant from the circumference, hence all radii of the ſame, or of equal circles, are equal. *See remark to definition* 14.

16. *The*

16. *The diameter*, of a circle, is a right line drawn from any point of the circumference to another, paſſing through the center,

as AD or BE. fig. 10.

The diameter of a circle, is therefore, equal to double the radius, and divides the circle, from its *paſſing through the center*, into two equal parts.

17. *A chord*, is a right line drawn in a circle from any point in the circumference to another, but *not paſſing* through the *center*,

as B G. fig. 10.

A chord, therefore, from its *not paſſing* through the center, divides the circle into two unequal parts, called *ſegments*; and the diameter is ſometimes called a chord, but, it is the greateſt chord that can be drawn in a circle.

18. *An arch* of a circle, is any part of the circumference,

as B F, or FBD, &c. fig. 10.

19. *A ſemicircle*, is one-half of the circumference,

as AFBD. fig. 10.

20, *A quadrant*, is one-fourth part of the circumference, or one-half of a ſemicircle,

as AFB, BGD. fig. 10.

21. *The complement of an arch*, is what any arch *wants* to make a *quadrant*,

rant, as AF or FB, are complements of each other. fig. 10.

22. *The ſupplement of an arch*, is what any arch wants to make a *ſemicircle*, as FBGD is the ſupplement of AF. fig. 10.

23. *A Segment of a circle*, is any part of the ſurface contained under any arch, and a chord joining its extreme points,
as GAGB or BFDEG. fig. 10.

24. *The ſector of a circle*, is that part of the ſurface. which is contained between two radii, and the arch which joins or ſubtends them
as BCF, or FCD. fig. 10.

OF ANGLES.

25. *A plane angle*, is the mutual inclination of two lines, meeting each other in a point on a plane, ſo as not to make one right line,
as ABC. fig. 11.

An angle is generally expreſſed by *three* letters, one at the angular point, or point of meeting, and the remaining two at the other extreme points of the lines: but, in expreſſing the angle, be careful always to read the letter at the angular point in the middle,
as

as ABC, or CBA. Sometimes, for brevity, an angle may be expreſſed by the middle letter only, as, the angle B: this can never be done, but when there is only *one* angle at *that point;* for, when more angles meet at one point, the three letters muſt be uſed. But the ſhorteſt way of expreſſing an angle is, by a letter placed within it, as *a*, (fig. 11) and obſerve, that if one plane angle be laid upon another, and the angular points and lines of both fall upon and touch each other, in that caſe they are ſaid to be equal: and, as the lines forming a plane angle, may be of the ſame, or of different kinds, therefore plane angles are denominated according to the lines which conſtitute them, as

26. *A right lined angle*, is that which is formed by right lines,
as ABC. fig. 11.

27. *A curved lined angle*, is that which is formed by curved lines,
as FGH, GIH. fig. 12 & 13.

28, *A mixed angle*, is that which is formed by the meeting of a right line and a curved one,
as DEF. fig. 14.

Theſe are the names of angles reſpecting their ſides; but, with reſpect to their quantity or meaſure, they are otherwiſe denominated.

29. *The meaſure of an angle*, is the quantity of an *arch* of a circle contained between the lines of the angle, the *center*

center of the arch being fixed at the *angular point;* as the arch *g h* is the meaſure of the angle *g a h.*

fig. 15.

As right lines are eſtimated in feet, yards, &c. &c. ſo is the quantity of an arch eſtimated in the parts of a circle, which are called *degrees*, *minutes*, *ſeconds*, &c. and, for this purpoſe, the circumference of every circle is, by mathematicians, divided into 360 equal parts, called *degrees;* each degree into 60 equal parts, called *minutes;* and ſo on, into ſeconds, thirds, &c. Now, if you deſcribe a circle *e g h*, with any opening of your compaſſes, upon the angular point *a* (fig. 15.) as center; ſo much of that circle *g h*, that is, ſo many degrees of it as are contained between the lines *b a* and *c a* of the angle, is ſaid to be the meaſure of the angle *g a h*. And here obſerve, that if the lines of the angle be produced to *k* and *i*, or further at pleaſure, you cannot, therefore, make the angle *b a c*, or the arch *g h*, its meaſure, either more or leſs. Hence, if upon the point *a*, as center, another circle *f b c d* be deſcribed, the arch *b c*, which falls between the lines of the angle, will not contain a greater or a leſſer quantity of degrees, minutes, &c. than the arch *g h;* but obſerve, a degree, minute, &c. of the arch *b c*, will be greater than a degree, minute, &c. of the arch *g h:* that is, if one degree of the arch *g h* will meaſure twelve inches, one degree of the arch *b c* will contain more than twelve inches, for it will be the three hundred and ſixtieth part of a greater circle.

Angles, with reſpect to their meaſure, are denominated according to the number of degrees they contain; as

30. *A right angle*, is that which is meaſured by a quadrant, or the fourth part of a circle,

as ABC. fig. 16.

Or

Or, to give this a more geometrical definition, it is that angle, which, if one of its ſides be extended, the line extended will, with the other line, produce an *equal* angle. And here obſerve, as the whole circumference of every circle contains 360 degrees, a right angle, or quadrant, its meaſure, muſt contain the fourth part of 360, viz. 90 degrees, and, becauſe a quadrant is half a ſemicircle, (defi. 20) a ſemicircle will contain 180 degrees: alſo, it will meaſure *two*, and the whole circle *four*, *right angles*. Hence, all *right* angles are *equal*, each being the fourth part of a circle.

31. *An acute angle*, is that which is leſs than a right angle, as DBC. fig. 17.

That is, when the arch which meaſures it is leſs than a quadrant, or 90 degrees.

32. *An obtuſe angle*, is that which is greater than a right angle, as EBC. fig. 18.

That is, when the arch which meaſures it is greater than a quadrant, or contains more than 90 degrees; but leſs than a ſemicircle, or 180 degrees, becauſe, in that caſe the lines of the angle, if ſuch could be, would be in the continuation of each other, and therefore become one line, or the diameter of a circle, as DBC. (fig. 18).

Acute and obtuſe angles are frequently called oblique angles, beſides otherwiſe denominated, according to their ſituations with reſpect to each other; but, before they can be defined, it will be highly neceſſary to explain

THE

THE DIFFERENT SPECIES OF LINES, RESPECTING THEIR POSITIONS TO EACH OTHER.

33. *A perpendicular line*, is that which falls upon, and meets another in any point of it, and by not inclining to it on either ſide, makes one or two right angles with it,

as BC. fig. 19.

Thus, the right line BC falling upon AD, and inclining not more to one part of it than another, but making the angles ACB and DCB on each ſide of it equal, is therefore perpendicular; and thoſe angles on *each ſide* of a perpendicular, are always *right ones*. Here take notice, with reſpect to the line to which any other line may be perpendicular, it matters not in what direction it is drawn, for AC and CD are as much perpendicular to BC, as the line BC is to each of them, or both together as one line. And further, obſerve, that no more than *one* perpendicular can ſtand upon the ſame point C in a right line; for, were it poſſible, they muſt fall on one ſide of CB, as they cannot, from this ſuppoſition, coincide or join with it: as for inſtance, in the direction of the line CG. Now, as this line CG inclines more to one point of the line AD than the other, it cannot therefore be *perpendicular*.

34. *An horizontal line*, or level, is that line, the extreme points of which inclines neither upwards nor downwards,

as, AD. fig. 19.

To make this appear more clear, ſuppoſe yourſelf ſtanding near the ſea, where no object as a hill, &c. would

would prevent your clearly viewing its ſmooth ſurface; in that caſe, if a line or ſtring be held tight between your hands, in ſuch a poſition before you that it would appear to coincide with, or every where touch, the upper ſurface of the ſea, where the ſky, by ſeemingly joining it, terminates your view, the line or ſtring would then be in an horizontal poſition; and, from this termination of your ſight being called the horizon, ſo is this, and all other lines drawn in the ſame direction, called horizontal lines.

35. *A plumb, or vertical line*, is that line which, being produced, would be perpendicular to, or make right angles with, an horizontal line,

as E P. fig. 20.

This line receives its name, from having a piece of lead or a plummet P, (fig. 20) ſuſpended by one of its ends, as, to the end of a ſtring or chord: this ſtring being hung by, or ſuſpended from the other end, the point of ſuſpenſion being from the pin E, or more properly the end of a wooden rule about three feet long and four inches broad, as E P in fig. 21; now, if this rule be held ſteady in a perpendicular poſition, in that caſe, the cord line will coincide with a right line E P, marked, or drawn thereon, from the point of ſuſpenſion E; hence this inſtrument received the name of a *plumb-rule*. Again, if this plumb-rule be made faſt, and fixed perpendicularly to, or at right angles with another rule, as A D, (fig. 22) both rules being thus connected, will form an inſtrument called a *level*; for by bringing the plumb-rule (EP) thereof to its proper and perpendicular poſition, you, at the ſame time, bring the rule A D, to which it is faſtened, to an horizontal one, and will therefore produce the level, or true horizontal line. Theſe two inſtruments are of ſuch ſingular uſe in mechanics, that many circumſtances depend upon them; as by the plumb-rule all buildings are made to ſtand erect; and, any

object

object being placed in the direction of this line, cannot fall ſo long as it continues therein; and by the latter, all planes, and veſſels holding water, and other veſſels are ſet level, and made to hold their full quantity; beſides many other uſeful purpoſes, as giving a regular fall to water, &c. &c. For ſome further uſeful obſervations on plumb-lines, ſee obſervation to def. 39, after parallel lines.

36. *An oblique line*, is that which is neither perpendicular, nor horizontal, but between both,

as C G. fig. 23.

Hence, acute and obtuſe angles are called *oblique angles*.

37. *A diagonal line*, is a line drawn acroſs any figure, from any of its angles to the oppoſite one,

as C G. fig. 23.

38. *A baſe line*, is the bottom line, or that ſide of any figure on which it is ſuppoſed to ſtand,

as C D. fig. 23.

And here obſerve, that any ſide of a figure, at pleaſure, may be made the *baſe*, for which purpoſe the longeſt is generally approved.

39. *Parallel lines*, are ſuch lines as are every where equally diſtant from, and no where inclined to, each other,

as AB & CD. fig. 24.

By

By being equally diſtant is to be underſtood, their diſtance from each other is every where the ſame; that is, were they produced infinitely both ways, (ſee fig. 24) they would never meet; but during their being ſo produced, would conſtantly preſerve this *equality* of diſtance. From which definition of parallel lines, it is evident, that if another right line croſs or cut them, they will both be equally inclined to that line, or it will make equal angles with them; as the angles E *d* B (fig. 25) and E *a* D will be equal. Again, if a line be perpendicular to one of them, it will alſo be perpendicular to the other; hence, the diſtance between two parallel lines is meaſured, for it is equal the length of a perpendicular between them; and if two or more perpendiculars be drawn between two parallel lines they will be equal, and parallel to each other, for, they will, each of them, make equal and right angles with the parallels. Wherefore, perpendiculars drawn between parallels, will intercept or contain, between their extremities, equal parts of the parallel lines; thus, in the parallels A B, C D, the parts, *a b* and *c d*, intercepted between the extremities of the perpendiculars, *a c* and *b d*, are equal. *

And obſerve further, that all perpendiculars to the ſame right line are parallel. Again, all horizontal lines are alſo parallel; for, a plumb-line croſſing them, would be perpendicular to each. But obſerve, with reſpect to plumb-lines, no two of them can be parallel, becauſe, they (different from all other lines) have a tendency to one and the ſame point, viz. the center of the earth, where, if nothing prevented their reaching it, they would certainly meet; therefore from the nature of parallel lines, cannot be parallel to each other. This inclination to that point, in the plumb-line, is owing to an attractive quality in all ſolid bodies; of which the plummet to this line, is one. This quality is called the *gravity*

* What is here obſerved of parallel *lines*, holds equally good with reſpect to parallel *plane ſurfaces*, cut in the ſame manner by other planes, for, all planes, viewed in a continuation of themſelves, will appear to be *right-lines*. See remark def. 5 & 8.

or

or *weight* of the body, whereby, it has a tendency downwards, or an inclination to fall, which it would certainly do, if not prevented, until it would reach the ground, or ſurface of the earth; and, was this motion downwards, to be no way prevented by the earth's ſurface, but continue; the falling body would *proceed onwards*, in a right-line, until it reached the *center* of the earth: that is, a plumb-line would, by the gravity of its plummet, be always in the direction of a right-line, conceived to be drawn from the earth's ſurface and continued to its center; wherefore, no two plumb-lines can be *parallel:* however, as this tendency of plumb-lines to each other on the ſurface of the earth, is ſo ſmall, as not to be ſenſibly diſcovered; they may, without any impropriety, be conſidered as parallel to each other; although, ſtrictly mathematically, they are not ſo.

40. *A tangent line*, is a right-line outwardly touching any figure, but not cutting it, as A B. fig. 26.

41. *Point of contact*, is that point in any figure, where the tangent line touches it, as C. fig. 26.

42. *A Secant line*, is that line drawn in any figure, ſo as to cut it, as C D. fig. 26.

OF TRIANGLES.

43. *A triangle*, is a plane ſurface bounded by three lines, as A, B, or ABC. fig. 27.

The *triangle*, is the moſt ſimple of all right-lined figures, it is expreſſed by one letter within it, as A, or B, (fig. 27) or, by three letters placed at the angular points, as ABC (fig. 27); but, here obſerve, that of the three lines forming a triangle, any two of them muſt be greater than the third; for, if it were not the caſe, the two ſhorter lines would never meet; conſequently, no triangle could be formed; hence, any two lines of a triangle, muſt always be underſtood, when taken together, to be greater than the third. 20 E. lib. 1.

Triangles, are denominated, either by the lines which conſtitute them; or, from the angles formed by them; hence, triangles are ſaid to conſiſt of ſix parts, viz. three ſides, *and* three angles; *wherefore, they have three names with reſpect to their ſides, and three with reſpect to their angles; as,*

44. *An equilateral triangle*, is that which hath *three equal* ſides,
as A. fig. 27.

45. *An iſoſceles or equicrural triangle*, is that which hath two of its ſides equal,
as B, DCE, DBE and DAE. fig. 28, & 29.

46. *A ſcalenum triangle*, is that which hath all its ſides unequal,
as B. fig. 27.

Theſe are the three names given a triangle, with reſpect to its ſides; *the three following, are in reſpect to its* angles, *as*

47. *A*

47. *A right-angled triangle*, is that which hath one of its angles a *right* one,

as A B C, fig. 30; which is right-angled at B.

Here obſerve, that the ſides conſtituting a right-angled triangle, are particularly denominated, from thoſe of any other, viz. the longeſt ſide, or that oppoſite to, or joining the ſides of the right-angle, is called the hypothenuſe; the other ſides are reſpectively named, one, the perpendicular, and the other the baſe.

48. *An acute-angled triangle*, is that which hath *all* its angles acute,

as A, or B. fig. 27.

49. *An obtuſe-angled triangle*, is that which hath one of its angles obtuſe,

as B C D, fig. 31; which is obtuſe angled at C.

Obſerve, acute *and* obtuſe-angled triangles, *are frequently called* oblique-angled triangles; *and further take notice*, a right-angled triangle, *may be either* iſoſceles, or ſcalenum; *and an* iſoſceles triangle, *may be either* acute, *or* obtuſe-angled; *as* D B E, (fig. 29) is *right-angled*, D A E *acute-angled*, and D C E *obtuſe-angled*, *yet each triangle is an* iſoſceles.

OF QUADRILATERALS.

50. *A quadrilateral figure*, or *quadrangle*, is a plane ſurface bounded by

four right-lines; it has *conſequently*, four angles,

as A. fig. 32.

A quadrilateral figure, the next in order to a triangle, is ſo called, from its being bounded by four lines or ſides, and is generally expreſſed by four letters, &c. (ſee remark to def. 10) it is differently denominated, according to the lines, and angles, which conſtitute it, as

51. *A parallelogram*, is a quadrilateral figure, which hath its oppoſite ſides equal, and parallel, and oppoſite angles equal,

as A, B. fig. 32, 33.

This is a general name for all quadrilateral figures, which have the above properties, viz. of parallel ſides, and equal oppoſite angles.

52. *A rectangle*, or *oblong*, is a quadrilateral figure, which hath *all* its angles *right ones*, conſequently equal,

as ABCD. fig. 34.

This term *rectangle*, is ſynonymous with the word *product* in arithmetic, which is a number produced, from the multiplication of one number by another, thus, 3 multiplying 4, will equal 12; which 12 in arithmetic is the product of 3 and 4; now, as theſe numbers 3, and 4, may be feet, yards, or any other lineal denomination at pleaſure; (ſee note to def. 6) they may therefore, geometrically be repreſented by two lines, one of 3, and the other of 4 feet, &c. Now, by placing theſe two lines perpendicularly to each other, and conceiving one of them, as CB, (fig.

(fig. 35) ſuppoſe of 3 feet, to move forward in the direction of the other AB, ſtill keeping parallel to itſelf, until it paſſes over a ſpace, equal in length to the other of four feet, &c. or, until it arrives at the point A; while, at the ſame time, let the other line AB of 4 feet, be conceived to move perpendicularly, and parallel to itſelf, through a ſpace equal to BC, *(for without conceptions of this kind, geometrical deſcriptions cannot be made with any propriety)* ſuch compound motion of the two lines, will form the rectangle of AB and BC; or the parallelogram ABCD, or BD*. Laſtly, if we ſuppoſe points were taken in each line, one foot diſtance from each other, theſe points, would, during the motion of the two lines, draw other lines croſſing each other, one foot diſtant; and divide the rectangle into 12 ſmall quadrilaterals, equal to each other, of one foot every way; which 12 rectangles expreſs the rectangle of a line of 3 feet multiplying a line of 4 feet; which is equal to the product before mentioned; and ſo of all other rectangles whatſoever.

53. *A Square*, is a parallelogram which hath four equal ſides, four right, and *conſequently equal* angles,

as A. fig. 36.

That is, a *ſquare* is both *equilateral* and *equiangled.*

54. *A rhombus*, is a parallelogram which hath four equal ſides, but angles oblique,

as B. fig. 37.

That is, a *rhombus* is *equilateral*, but not *equiangled.*

* Any of which expreſſions will anſwer, (ſee def. 10.)

55. *A*

55. *A Rhomboides*, is a parallelogram, which, hath only its *oppoſite* ſides equal and parallel, and angles oblique,

as C. fig. 38.

That is, a rhomboides is *neither* equilateral nor equiangled.

56. *The complements* of any *parallelogram*, are two parallelograms adjoining to two others about its diagonal,

as GO and OC. fig. 39.

That is, ſuppoſe you take any point, in the diagonal of a parallelogram, as O, and through that point, let the lines BF, and HD be drawn, parallel to the ſides CE, and AC, theſe lines, will divide the parallelogram into four others, *a*, *b*, *c*, and *d*, all meeting in the point O; two of which, *a* and *c*, are ſaid to be about the diagonal, as it paſſes through them; and the other two *d*, and *b*, are the *complements* of thoſe about the diagonal, to the whole parallelogram AE: and here, it will be proper to obſerve, that the diagonal line of any parallelogram, is frequently called a *diameter*; for if two diagonals be drawn in any parallelogram (as in fig. 34.) they will paſs through the middle point E, where they interſect each other: and, as a diameter divides a circle into two equal parts, in the ſame manner the diagonal of a *parallelogram*, divides it into equal parts.

57. *The Gnomon* of a parallelogram, is the complements of the parallelogram, taken together with either of the parallelo-

rallelograms, about the diagonal;
as the parallelograms *d*, *a*, *b* or *d*, *c*, *b*.
fig. 39.

58. *A trapezium*, is an irregular quadrilateral figure, having neither its ſides, nor angles equal,
as D. fig. 40.

That is, every quadrilateral figure, which is not a rectangle, or parallelogram, is a trapezium.

Having now defined the moſt uſeful parts of three and four ſided figures; I might here proceed with the figures of more *than* four *ſides: but, before that can be done, it will be neceſſary to mention ſome other uſeful particulars, concerning* angles *and* lines, *which could not be ſo well underſtood before, and to which I refered, as*

59. *The vertex*, or top, of any figure, is the moſt remote point of it from the baſe,
as A. fig. 41.

The word *vertex*, is more generally applied to triangular figures; for, as it more properly ſignifies a point, than the top of any figure, it cannot therefore be ſo well applied to quadrilaterals; becauſe, it might be argued, that the line CA (fig. 42) is the top of the quadrilateral ACDE; whereas, no more of that line, than the higheſt point A, can be conſidered as the vertex: and obſerve, that in quadrilaterals, &c. the vertical point, will be always that point, from which the longeſt perpendicular can be drawn to the baſe.

60. *The altitude*, or *heighth* of any figure, is the length of a line drawn perpen-

perpendicularly from the *vertex* to the *baſe*,

as AB. fig. 41, 42.

61. *The external angle* of any figure, is that angle, which lies at the outſide of it, formed, by producing one of its ſides,

as A. fig. 43.

62. *An internal angle*, is any angle within the figure,

as *a*. fig. 43.

63. *An adjacent angle*, is an angle immediately next to, or adjoining another angle, having one of *its ſides*, a ſide of *the other*,

as A is adjacent to *a*, or *a* is adjacent to A, &c. fig. 43.

Note, this ſide which ſtands between two adjacent angles, is called a common ſide, *becauſe it is a ſide to both.*

64. *A common*, or *intermediate angle*, is an angle between two others, adjacent to both, and may occaſionally be added to either,

as *a*. fig. 44.

That is, the angle *a*, ſtanding between the angles *b* and *c*, may be joined to either; thus, by joining it to

to *b*, you have the angle BAE; again, by joining it to *c*, you get the angle CAD; in ſuch caſes, were the angles *b* and *c* equal, the common angle *a*, being added to each, would make the enlarged, or increaſed BAE, and CAD, equal.

65. *A common*, or *intermediate arch*, is the ſame as a common angle, it being the meaſure of it,

as DE is common to the arches BE and CD. fig. 44.

What was obſerved before of a common angle, may here, be applied to a common arch.

66. *A ſegment of a line*, is a part cut off from it, by another line,

as AC or CD are ſegments of the line AD. fig. 45.

67. *A common* or *intermediate ſegment* of a line, is that part of it which lies between, and adjacent to, the remaining parts of it,

as BC. fig. 45.

And obſerve that, (as in a common angle) where the remaining or adjacent ſegments are equal, the common ſegment BC being added to both, their ſum, or the lines AC and BD, will be equal.

68. *A ſubtend*, or *ſubtenſe*, is any ſide of a triangle, ſtanding oppoſite to one of its angles, whereby, it is ſaid to *ſub-*

ſubtend the angle to which it is oppoſite,

as E D is the ſubtenſe of the angles O or C. fig. 46.

69. *Alternate angles*, are thoſe which lie between two parallel lines, but, on the oppoſite ſides of a line cutting them,

as *a*, *c* and *d*, *b*. fig. 47.

That is, *a*, is *alternate* to *c*, and *b*, *alternate* to *d*, for they are on oppoſite ſides of the line E F, cutting the parallels A B and C D. And obſerve, with reſpect to the other angles about the parallel lines, they are denominated according to their *ſituation*; as *f*, is an *external angle*, and ſaid to be on the ſame ſide of the cutting line with *c*; which angle *c*, with reſpect to *f*, is called an *internal angle*. Obſerve the ſame with reſpect to the angles *e* and *d*. And laſtly, the angles *a*, *d*, *c* and *b*, are called internal angles, of which, *a* and *d* are ſaid to be on the ſame ſide, as alſo *c* and *b*.

70. *Vertical, or oppoſite angles*, are thoſe which touch at their *vertices*, or have their lines in continuation of each other,

as *f* is vertical to *a*, and *e* to *b*. fig. 47.

71. *An angle at the center*, is that which is contained between two radii in a circle,

as E O D. fig. 46.

72. *An*

72. *An angle in a ſegment*, is that which hath its angular point, in the arch of the ſegment, and its lines terminating in the extremities of the chord of the ſegment,

as ECD. fig. 46.

73. *The angle of incidence*, is that which is formed by an *oblique line*, and a *perpendicular* falling on the ſame point in a right-line,

as CBD. fig. 48.

This angle is produced, by ſuppoſing any body, as C, to be put in motion, in an oblique direction, as the line CB, until it ſtrikes the line AF, in the point B; where, becauſe it meets with reſiſtance, it will fly off in the direction of the oblique line BE; then, in ſuch caſe, if the perpendicular BD, be raiſed from B, where the body ſtrikes the line AF, the angle formed between CB, and BD, viz. CBD, is called the angle of incidence; this angle, is by ſome, ſuppoſed to be contained between the oblique line, and the line it falls on, as CBA.

74. *The angle of reflection*, is that which is formed between a *perpendicular* to a right-line, and an *oblique line*, flowing from the point on which it ſtands,

as DBE. fig. 48.

This angle is the *converſe* of the angle of *incidence*, it being cauſed from the body C, reflecting from the line AF, which it ſtrikes in the direction of the line BE, it is always eſteemed *equal* to the angle of inci-

incidence; becauſe, it reflects, or flies off, in the ſame angle with AF; conſequently muſt have the ſame inclination to the perpendicular DB, or to the line AF.

75. *Similar, or proportional figures*, are ſuch, as have all the angles in one figure, *equal reſpectively* to thoſe of the other, and *that*, whether the ſides be equal or not,

as AB. fig. 49.

Thus, if you lay one figure upon the other, ſo that any angle in one, ſhall fall upon its equal angle in the other; in that caſe, all the remaining ſides of the figures, will become parallel to each other. The dotted lines ſhew the correſponding figures, when laid upon each other.

76. *Congruous figures*, are ſuch as agree, and correſpond with each other in every reſpect, ſide to ſide, and angle to angle,

as B, C. fig. 50.

Thus, ſuppoſe one figure B, laid upon the other C, if in that ſituation, the ſides, and angles of both, fall upon, and agree, ſo that they every where touch, or coincide with each other; they are ſaid to be congruous figures; and, from their thus agreeing with each other, they are ſaid to be equal.

OF THE LINES DRAWN IN A CIRCLE.

77. *The right ſine of an arch*, is a right-line, drawn, from one extremity of

of the arch perpendicular to a radius, or diameter drawn to the other end of the arch,

as AB, or its equal GC. fig. 51.

The ſine, is always equal to half the chord, of double the arch, thus, the arch BI, being double BD; half the chord BI, viz. BA, will be the right ſine of the arch BD, which is half the arch BI.

This, and the ſeven following definitions, more properly belong to trigonometry *than* geometry; *but, as I intend to ſhew the method of drawing them, in the practical part of this work, I therefore mention them here, that the reader may be acquainted with them, when he comes to deſcribe them.*

78. *The co-ſine*, or *ſine complement*, of an arch, is the *right-ſine*, of the complement of the arch,

as GB, or its equal CA. fig. 51.

79. *The tangent* of an arch, is a right-line, perpendicular to the radius of the arch, *outwardly* touching it, and terminated by a line drawn from the center of the arch, through the other end,

as DE. fig. 51.

80. *The co-tangent*, or *tangent complement* of an arch, is the tangent of the complement of the arch,

as FH. fig. 51.

81. *The*

81. *The ſecant line*, is a right-line drawn from the center of the arch, through its extremity, meeting the end of the tangent, drawn from the other extreme,
as CE. fig. 51.

82. *The co-ſecant*, or *ſecant complement* of an arch, is the ſecant, of the complement of the arch,
as CH. fig. 51.

83. *The verſed ſine* of an arch, is that part of the radius of the arch, contained between the tangent, and the point where the right-ſine falls upon it,
as AD. fig. 51.

84. *Co-verſed ſine*, is the verſed ſine of the complement of the arch,
as GF. fig. 51.

Here obſerve, from the laſt two definitions, that the co-ſine CA, *and* verſed ſine AD, *added together; alſo, the* ſine GC, *and* co-verſed ſine GF, *added together, will equal radius.*

OF POLYGONS.

85. *A Poligon*, or *many-ſided* figure, is that which hath more than four ſides, and is either regular, or irregular.

86. *Re-*

86. *Regular*, or *ordinate polygons*, are ſuch as have all their ſides and angles equal,

as ABCDE, or *a b c d e.* fig. 52.

That is, all the angular points of any regular polygon, would fall into the circumference of a circle, was it drawn round it. Alſo, if a circle was ſwept within it, the *ſides* of the polygon would become reſpectively *tangents* thereto. Polygons are variouſly denominated, according to the ſides which conſtitute them; of which, thoſe that follow are the moſt principal:

As, if of five ſides, a *pentagon*
of ſix, - - an *hexagon*
of ſeven, - an *eptagon*
of eight, - an *octagon*
of nine, - - a *nonagon*
of ten, - - a *decagon*
of eleven, an *undecagon*
of twelve, - a *dodecagon*
of fifteen, - a *quindecagon*, &c.

to infinity; there being no end to the number of ſides a regular polygon may contain; to enumerate all of which would be endleſs. But obſerve, the greater number of ſides any *ordinate polygon* contains, the nearer will the polygon approach in form to that of a *circle*: for in that caſe, the ſides of the polygon would differ very immaterially from the circumference of a circle drawn round it; hence, Mathematicians draw this concluſion, that a *circle* may be conceived to be a *polygon* of an infinite number of ſides, from which inference they attempt to make a *ſquare* ſurface, *nearly equal* that of a circle, whereby they meaſure its ſuperficial content. (*See prob.* 62.)

87. *The perimeter*, or *periphery* of any polygon, is the ſum, or aggregate of all the ſides of the polygon, taken together.

88. *A*

88. *A polygon*, or any *figure*, is ſaid to be inſcribed in a circle, when all its angles touch the circumference;
as the polygon *abcde*, is inſcribed in the circle ABCDE. fig. 52.

89. *A circle* is ſaid to be *inſcribed in a polygon*, or, a *polygon* is ſaid to *circumſcribe a circle*, when each ſide of the polygon touches the circumference,
as, the polygon ABCDE circumſcribes the circle within it, for every ſide of it touches the circle. fig. 52.

90. *Concentrick circles*, *polygons*, &c. are all ſuch as have the ſame, or common center.

Thus, the circle and polygons in fig. 52, are ſaid to be concentrick, as they all have the ſame *center* F.

91. *Excentrick circles*, *polygons*, &c. are all ſuch as *have not* the ſame center.

This will be plain, if you conſider the former definition of concentrick circles. But, this term *excentrick* is more generally applied to eliptical or oval like figures (ſee *prob.* 67) which, are ſaid to be more or leſs excentrick as the centers are more or leſs diſtant.

92. *The direct radius* of any ordinate polygon, is a right-line drawn from the center of the polygon, perpendicular to any of its ſides, as F*g* or FG. fig. 52.

93. *Super-*

93. *Superficial content* or *area* of any figure, is the number of ſquare feet, yards, acres, &c. contained in it.

That is, the number of ſquares (the ſides of which ſhall be one foot, yard, &c.) contained in the figure, will be its area in feet, yards, &c. (*See obſervation to def.* 52.) where the area of the rectangle A B C D, (fig. 35.) was ſhewn to be equal to 12 ſmall ſquares, which ſquares may be eſtimated in feet, yards, &c. as their ſides were firſt conſidered, or taken in any of theſe meaſures.

94. *To biſect, triſect, quadriſect, &c.* any line, arch or angle, is, to cut it into two, three, four, &c. equal parts.

95. *Any figure* is ſaid to be *deſcribed*, when the lines, circles, &c. belonging thereto, or neceſſary for its formation, are drawn.

96. *The extremes* of any figure, are the utmoſt boundaries, or limits thereof.

Thus, the extremes of a *line* are points; (def. 5.) of a *ſurface*, lines; (def. 7.) of a *ſolid*, ſurfaces; (def. 11.) but obſerve, a circle, and a ſphere, or globe, cannot be ſaid to have *extremes*; the former being bounded by a curved line, which, revolving *into itſelf*, leaveth no extreme, but one bounding line; and the latter in the ſame manner, by a *curved ſurface*.

OF PROPORTION.

97. *Proportion*, is the ſimilitude of two, or more ratios.

98. *Ratio*, is the compariſon of one number or quantity, with another; it conſiſts of two terms, called the *antecedent* and *conſequent*.

99. *The antecedent*, is the *compared*, or firſt mentioned term of the *ratio*.

100. *The conſequent*, is the following or *laſt mentioned* term of the ratio.

A ratio, is expreſſed or written down, by placing the antecedent *before* the conſequent: thus, A : B, which letters A and B, may repreſent *any* quantity; and the ratio is to be read thus, as A is to B, the mark (:) between the letters ſignifying the words (*is to*). A ratio is frequently expreſſed, by placing the antecedent *over the* conſequent, thus $\frac{\text{A } \textit{antecedent}}{\text{B } \textit{conſequent}}$, which expreſſion is read the ſame as the former; and from this ſituation of the terms over each other, we are to learn, that, when terms are in a ratio, the antecedent muſt be *divided by* the conſequent; and, as the *line* between them *denotes diviſion*, the quotient thence ariſing, is called the exponent of the ratio.

101. *The exponent of a ratio*, is the *quotient* ariſing from dividing the antecedent by the conſequent.

Thus,

Thus, in the ratio of A : B, or of 6 : 24, (fig. 53) the exponent would be one-fourth; but the *exponent* of the ratio of B : A, or of 24 : 6, would be 4: that is, in the former example, the *antecedent* contains but one-fourth of the *conſequent;* and in the latter, the antecedent contains its conſequent 4 times, &c. and obſerve, in expreſſing any ratio, the firſt method will be found more convenient, than by placing the terms over each other, as it takes up leſs room.

102. *One leſſer quantity, is ſaid to meaſure another greater quantity*, when the greater contains the leſſer, a certain number of times exactly.

Thus, the quantity A, equal 6; is ſaid to meaſure the quantity B, equal 24; (fig. 53) becauſe B, or 24, contains A, or 6, exactly 4 times.

103. *A common meaſure*, is any one *leſſer* quantity, which will meaſure or divide two or more *greater* quantities, without leaving any remainder.

Thus, A is a common meaſure to B and C, (fig. 54) for B, contains A, four times; and C, five times, exactly.

104. *Commenſurable quantities*, are thoſe which have a common meaſure.

Thus, B, equal 24, (fig. 53) is a commenſurable, quantity becauſe A equal 6, meaſures it exactly, without leaving any remainder.

105. *Incommenſurable quantities*, are thoſe which cannot be meaſured by any

other quantity greater than *unity*, or one.

That is, when no determined quantity, greater than an unite can be found, which, being taken or multiplied, any number of times, will equal it.

Any two quantities, whoſe ratio, or proportion to each other, can be expreſſed by commenſurable quantities, are ſaid to be commenſurable to each other; but, when their ratio cannot be ſo expreſſed, they are ſaid to be incommenſurable to each other, or, the ratios are called commenſurable, or incommenſurable, accordingly.

106. *A multiple*, is a greater quantity which contains a leſſer, a certain number of times without a remainder.

Thus, B (fig. 53) is a multiple of A, becauſe it contains A four times *exactly*.

107. *Like multiples*, are thoſe quantities, which contain their parts equally.

Thus, A and C (fig. 55) are like multiples of B and D, for A contains B, as often as C contains D; or, to make this more plain, inſtead of the letters or lines, uſe the *numbers* annexed to them: thus, 18 and 12, are like multiples of 6 and 4; becauſe, 18 contains 6, as often as 12 contains 4, viz. 3 times: but in thus uſing the numbers, be careful to have reference to the line which repreſents that number, as *lines* are more properly the buſineſs of geometry than *numbers*.

108. *An aliquot part*, is an even part of any quantity.

Thus, A is an aliquot part of B, (fig. 53) for B, or 24, contains A or 6, exactly four times, without leaving any remainder.

109. *An*

109. *An aliquant part*, is an uneven part of any quantity.

Thus, D is an aliquant, or *uneven part* of A (fig. 56) for A does not contain D, any certain number of times.

110. *Like aliquot*, *like aliquant*, or *like parts*, are ſuch, as are equally contained in their wholes.

Thus, B and D (fig. 55) are like aliquot parts of A and C; for, B is as often contained in A, as D is contained in C, that is, four times. In the ſame manner, A and B (fig. 56) are like aliquant parts of C and D, they, being equally contained in C and D, *viz.* once and one-fixth.

111. *Equal ratios*, are thoſe whoſe exponants are equal, or whoſe antecedents are like multiples, or like parts of their reſpective conſequents.

Thus, the ratio A : B, is equal the ratio C : D (fig. 56) for A is the ſame, or like multiple of B, that C is of D. (*Def.* 107).

112. *Ratio of equality*, is, when in any ratio the antecedent and conſequent is *the ſame*, or, the exponent is unity.

Thus, if one quantity A (fig. 57) be equal to another quantity B, then the ratios A : B; A : A; and B : B are the ſame; for, in each ratio, the exponent is unity or 1 (def. 101) or, the antecedents and conſequents are equal.

113. *Four quantities or magnitudes are ſaid to be proportional*, when, by comparing them together, two by two, they will be found to produce *equal ratios*.

Thus,

Thus, ſuppoſe the magnitudes * A, B, C and D, (fig. 56) be proportional, by comparing them, *firſt*, thus A : B :: C : D, you will find in the ratios of A : B and C : D, that the antecedents, A and C, are like multiples (107) of their reſpective conſequents, B and D; therefore, the ratios A : B and C : D are equal, (111) and this proportionality is thus read, *as* A *is to* B, *ſo is* C *to* D. *Secondly*, by comparing them thus, A : C :: B : D, the ratios A : C and B : D are equal, (111) for the antecedents, A and B, are like aliquant parts (110) of their reſpective conſequents, C and D. *Thirdly*, compare them thus, D : C :: B : A, or B : A :: D : C, in either caſe the antecedents, D and B, will be like aliquot parts (110) of their reſpective conſequents, and the ratios in each caſe equal, therefore the four quantities are proportional. And obſerve, from the different methods of comparing quantities in proportion, ariſe different kinds of ratios, as

114. *Direct ratio*, is, when in *four* proportional quantities, the ratio of the *firſt* and *ſecond* terms, is equal to the ratio of the *third* and *fourth*,
thus A : B : : C : D. fig. 58.

That is, when the antecedent is compared with the conſequent, (fig. 58) or the terms of the ratio keep their *direct* order.

115. *Alternate natio*, is, when in four proportional quantities, the ratio of the *firſt* and *third* terms, is equal to the ratio of the *ſecond* and *fourth*,
as A : C : : B : D. fig. 58.

* In def. 4, a *line* was not there conſidered as a magnitude in itſelf; but, as a line from its length can be expreſſed in number, it is, therefore, generelly underſtood by Mathematicians to be a magnitude, from its being capable of extenſion and diminution. Obſerve the ſame with reſpect to a *ſurface*, (*def.* 7) which may be conſidered as a magnitude for the ſame reaſon.

That

That is, when antecedent is compared with antecedent, and conſequent with conſequent.

116. *Inverſe ratio*, is, when in four proportional quantities, the ratio of the *ſecond* and *firſt* terms, is equal to the ratio of the *fourth* and *third* terms, as B : A : : D : C or D : C : : B : A, fig. 58.

That is, when the conſequent is put before the antecedent, and in that manner compared with it, whereby the terms of the ratio become changed or *inverted*.

117. *Geometrical ratio*, or *geometrical proportion*, is, when the terms in any proportion regularly increaſe, by a common multiplier, as 2, 4, 8, 16, 32, &c. or decreaſe by a common diviſor, as 32, 16, 8, 4, 2, &c. and preſerve equal ratios.

That is, the terms 2, 4, 8, 16, &c. increaſe by continually multiplying each term by 2, thus, 2 *the firſt term*, multipled by 2, produces 4, *the ſecond term*, &c. where, the ratios, 2 : 4 ; 4 : 8 ; 8 : 16 ; &c. are equal (def. 111). Again, in the terms 32, 16, 8, &c. they decreaſe, by continually dividing by 2 ; thus, 32 divided by 2, gives 16, &c. in which terms the ratios 32 : 16 ; 16 : 8, &c. are equal (def. 111). In all geometrical proportions, when the terms are even, the *rectangle* or product of the *extremes*, viz. (the firſt and laſt,) will equal the rectangle of any two *means* (or middle terms) equally diſtant from thoſe extremes. Thus, in the *ſeries* *

* This is a name given to all numbers, in geometrical or *any other* proportion, taking, in idea, all the terms of the proportion together.

2, 4,

2, 4, 8, 16, 32, 64, the product of the *laſt* term 64, and the *firſt* term 2, will equal the product of 32 multiplying 4; alſo, to 8 multiplying 16, viz. 128. Again, if the terms of the ſeries be odd, not only the above property will ſtand good, but alſo the *rectangle* of the *extremes* will equal the *ſquare* * of the *mean* or middle term; thus, in the progreſſion, 2, 4, 8, 16, 32; 32 multiplying 2, equal 64, which is alſo the product of 4 multiplying 16, and of 8 multiplying 8, that is, the ſquare of the middle term. On this property of geometrical quantities, does the reaſon for the management of the terms, given in the *rule of three*, in arithmetic depend; where you have *three* terms given to find a *fourth*; to perform which, the general rule is, "*Multiply the ſecond and third terms together, and divide by the firſt.*" This muſt be the only method, becauſe, in the three given terms, you have (of four terms in proportion) the two means, (that is the *ſecond* and *third* terms,) and one extreme (the *firſt* term) given, to find the other extreme, (that is, the *fourth* term.) Now, as the product of the two *extremes*, equal the product of the *means*, from what has been before ſhewn, you muſt therefore, firſt find the product of the two means, that is, *multiply the ſecond and third terms together*, and this product muſt be divided by the given extreme; that is, *divide by the firſt term*, and the quotient will evidently be the other extreme, or fourth term. For, in diviſion, the diviſor multiplying the quotient, will always *equal* the dividend; but, the dividend, in this caſe, is the product of the *ſecond* and *third* terms; therefore, when you divide by the firſt *term*, the quotient muſt be the fourth.

I hope, I ſhall be excuſed for intruding thus far, on my readers time, and patience, by introducing any thing, purely arithmetical, in a book of geometry; but when I conſidered, this rule was never explained by any author, on arith-

* By the term *ſquare* here mentioned, you are to underſtand any term multiplied by itſelf, as 4 multiplying 4, equal 16, which number 16 is the ſquare of 4, &c. all numbers produced in this manner, are ſaid to be ſquare numbers, and the number which produces the ſquare, is called the root, or radix of the ſquare.

methic,

methic, that I ever read, I therefore took this liberty, of ſatisfying ſuch of my readers, as may be ignorant of the reaſon for that rule, which they have hitherto taken for granted, as true; becauſe, (I ſuppoſe) they have always found it to anſwer, the deſign of the queſtion; if I have been ſo happy as to clear up this matter, to thoſe that want it, I am the more inclined to think thoſe who are acquainted with it already, will excuſe me, and paſs it over.

118. *Geometrical ratio continued*, is, when in ſeveral geometrical proportionals, it will be, as the *firſt* term is to the *ſecond*, ſo is the *ſecond* to the *third*, ſo is the *third* to the *fourth*, &c. or as A : B : : B : C : : C : D : : D : E, &c. fig. 59.

That is, the product of the firſt and third terms of ſuch proportionals, will equal the ſquare of the ſecond or middle term; thus 2, 4, 8, are terms in continued proportion, the compariſon of which will be as 2 : 4 : : 4 : 8; where 2 multiplying 8, will equal 4 multiplying 4, that is, to 16, the ſquare of the middle term; and ſuch proportionals are called continued, becauſe, the conſequent of the firſt ratio is *repeated*, and made the antecedent of the ſecond, whereby it is *continued* through the terms.

119. *Duplicate ratio*, is the proportion of ſquares.

That is, in a ſeries of geometrical proportionals, the firſt term is to the third, in a duplicate ratio of the firſt, to the ſecond, or, as the *ſquare* of the firſt, to the *ſquare* of the ſecond; thus, in the proportionals 2, 4 and 8 the duplicate ratio is, as 2 : 8 : : the ſquare of 2, to the ſquare of 4; that is, as 4 : 16, where the ratio of 2 : 8, is duplicate of 2 : 4; for, 2 : 8 : : 4 : 16, &c.

120. *Arith-*

120. *Arithmetical progreſſion*, is, when the terms in the progreſſion, increaſe or decreaſe by *equal* differences,

as 1, 2, 3, 4, or 2, 4, 6, 8, &c. are terms increaſing, &c. 8, 6, 4, 2, or 5, 4, 3, 2, 1, are terms decreaſing.

What has been obſerved, in regard to the properties of terms, in geometrical proportion, may be applied here, with reſpect to the properties of terms in arithmetical progreſſion; but, inſtead of the word *product* or *rectangle* there uſed, uſe the word *ſum* *, and inſtead of the word *ſquare*, uſe the word *double*.

More need not be ſaid here of arithmetical progreſſion, as it does not ſo properly belong to geometry; but, as it occured to me, and the inſerting it cannot be of any diſſervice, I took the liberty of introducing it, as it may ſometimes appear in the courſe of a work of this kind, and the reader be ignorant of it.

121. *A poſtulate*, is a ſimple truth, which may be eaſily granted without a proof.

As, that a right-line may be drawn from one point to another, &c.

122. *An axiom*, is a *ſelf-evident truth*, to which, any one may aſſent, on hearing it propoſed.

* This word ſignifies the total, of two or more terms added together, as, the ſum of 2 and 4 is 6, &c,

As,

As, that every *whole* line is greater than a *part* of it.

123. *A propoſition*, is the propoſing of ſomething to be done, and requiring a ſolution, or anſwer.

Propoſitions are of two kinds, *problems* and *theorems*.

124. *A problem*, is a practical propoſition, wherein ſomething is required to be *made* or *done*.

As, if it were required on a given line, to make an equilateral triangle, &c.

125. *A theorem*, is a ſpeculative propoſition, wherein, ſomething is propoſed or affirmed to be true, and requires a proof that it is ſo.

As, ſuppoſe it was declared, that if any right-line falls upon another, *it will make the angles on each ſide of it, together, equal to two right-angles*, which, becauſe in geometry, no declaration of this kind can be taken as true, it therefore, before it can be believed, requires that it muſt be *proved* to be ſo; for, until that is done, no affirmation in the propoſition, can be depended upon.

126. *A corollary*, is a concluſion drawn from a preceding propoſition.

As, if one right-line falling upon another, will make angles equal to two right ones; the *corollary* to be drawn from that propoſition, will be, that if ever ſo many right-lines fall upon another right-line in the ſame point, all the angles together will make only two right ones.

127. *A*

127. *A ſcholium*, is a remark made upon ſome propoſition, exemplifying the matter it contains.

128. *Hypotheſis*, is that part of a propoſition, which is *known* or *granted*.

129. *Theſis*, is that part of a propoſition, which is required to be proved.

Thus, if two right-lines are ſaid to croſs each other, the vertical, or oppoſite angles will be equal. In this theorem the *hypotheſis* is, that the two given lines are two right-lines, and cutting each other; and the *theſis* or matter to be proved is, that the vertical, or oppoſite angles *are equal*: the word hypotheſis has ſometimes been miſtaken for the word *data*, whereas, they are of different meanings; the word *data*, ſignifying that part of a propoſition, which is *given* or allowed to be true; and the *hypotheſis*, that which is *granted*, or ſuppoſed to be ſo.

130. *A lemma*, is but another name for a propoſition, therefore, is of the ſame import.

However, other writers, on this ſubject, may conſider this as a different term from a propoſition; I cannot agree with them, for ſuch diſtinctions only ſerve to confuſe our ideas, (which in geometrical concluſions ſhould be quite clear) for, they ſuppoſe a lemma neceſſary to prepare the mind, for the more clearly underſtanding a following propoſition; whereas, I ſee no reaſon why, in geometry, it ſhould not be as much a propoſition as any other; therefore, to make a difference, is quite unneceſſary; however, proper this diſtinction of terms may be in other mathematical works, in *Geometry*, it is not allowable.

131. *A*

131. *A converſe propoſition*, is that which declares in its affirmative part, every thing contrary to a proceeding one.

As, were it in any propoſition declared, that the *greateſt angle*, in any triangle, is oppoſite to the *greateſt ſide*, the converſe of which propoſition will be, that the *greateſt ſide* of any triangle would be oppoſite to the *greateſt angle*.

132. *By ſuppoſition*, in geometry, we are to underſtand, that a line, angle, &c. may, for the ſake of argument, be ſuppoſed to be drawn in any figure, or, that the propoſition may be otherwiſe than it is, whether is really ſo or not.

For, in demonſtrating any propoſition, it will ſometimes be neceſſary to ſuppoſe, ſuch, and ſuch lines, angles, &c. to be drawn, which, in reality, are not ſo; and by the abſurdity of the concluſions, which may be drawn from ſuch ſuppoſitions, the demonſtration is made evident; or, where a ſelf-evident propoſition is required to be proved, the only proof in ſuch a caſe, will be, to deny the affirmation of the propoſition, and ſuppoſe it to be different from what it is declared to be; whereby, on the abſurdity of ſuch ſuppoſition (it not being poſſible to prove it any thing otherwiſe, than what it was originally affirmed to be) we conclude it muſt be true; as, were it required to be proved, "*that every whole number, or quantity, is greater than its part;*" which being ſelf-evident, the only proof that can be brought, will be, to ſuppoſe that the whole number cannot be *greater*, but *leſs* than its part; and by examining the propoſition on this ſuppoſition, and finding an abſurdity appear, we therefore conclude the propoſition to be true. And this mode of proving any propoſition, is called

argu-

argumentum (or reductio) ad abſurdum; that is, the propoſition is abſurd or impoſſible on that ſuppoſition; therefore, what was affirmed, or declared of it, muſt be true.

133. *Conſtruction* of any figure, is the drawing or diſpoſing of its lines, angles, &c. geometrically, ſo as to make the proof of it more clear and evident.

And, in the conſtruction of any figure, care muſt be taken, to uſe only ſuch lines, and angles, as are already well underſtood, or the properties of which, have been previouſly proved.

134. *Demonſtration*, is the proving, or making clear to the underſtanding, whatever is affirmed in any propoſition, leaving you perfectly ſatisfied, as to the truth of it.

The demonſtration is generally performed, from having the propoſition firſt ſtated, and afterwards drawing ſuch lines, as may appear neceſſary for the proof; from the properties of which lines, (already known) the proof, or demonſtration thereof, becomes obvious and clear, and therefore, it is ſaid to be demonſtrated. The uſual methods of demonſtration, is by ſyntheſis, and analyſis, called the *ſynthetical*, and *analytical* methods.

135. *The ſynthetical method* of demonſtration, is, when we purſue the truth, chiefly by drawing reaſons from principles before eſtabliſhed, and propoſitions formerly proved; and thus, proceed

ceed from a regular chain of arguments, or, as it were, by regular ſteps or gradations, until we come to the concluſion.

This mode of argument, is called *compoſition*, and is in oppoſition to the analytical method; which latter method (although I ſhall not uſe it) I will however explain, becauſe, it is ſometimes miſtaken for the ſynthetical.

136. *The analytical* method, is the ſolving any theorem, by enquiring into the bottom, or fundamental principles of the propoſition: or, as it were, *taking it all to pieces*, and reſolving it into its parts; and by *puting them together* again, we ſee into the reaſon and nature of it.

This mode of argument is called *reſolution*, becauſe it reſolves any propoſition into its component principles, or parts.

AN

EXPLANATION

OF THE

CHARACTERS AND OBSERVATIONS USED IN THE FOLLOWING WORK.

+ *Plus* or *more*, is the ſine of *addition*, and any quantities between which it is placed, expreſſeth the *ſum* of the quantities; thus, $a + b$, or (*a plus b*) ſignifies the *ſum* of any quantity repreſented by *a*, *added* to the quantity repreſented by *b*; or, to make this better underſtood by a reader unacqainted with joining of ſuch quantities, ſuppoſe *a* repreſented the No. 15, and *b* the No. 5, then, would $a + b$, be the ſame as $15 + 5$, (15 *plus* 5), or 20, the *ſum* of 15 and 5, &c.

— *Minus* or *leſs*, is the ſign of *ſubtraction*, and any quantities between which it is placed, expreſſes the *difference* of the quantities; or, that the latter, is to be *ſubtracted* from the former; thus, $a - b$, or, (*a minus b*) ſignifies the *difference* between *a* and *b*; thus, ſuppoſe *a* and *b*, repreſented the ſame quantities as before, then $a - b$,

or

or (*a minus b*) would be the ſame as 15 — 5, or 10, the *difference* between 15 and 5, &c.

× *Into*, or the ſign of *multiplication*; and, when placed between any two quantities, it ſignifies the *product* of the quantities; thus, *a* × *b*, or (*a multiplying b*) expreſſes the *product* of *a* and *b*, or, if *a* equal 15, and *b* equal 5, as before; then *a* × *b*, will equal 15 × 5, or the *product* of 15 and 5, viz. 75, &c. *Again*, 4*a*, or 3*b*, ſignifies, that the quantity repreſented by *a*, viz. 15, is to be *multiplied* by 4, or taken 4 times, and that by *b*, 3 times; thus, 4*a*, equal 4 × 15, or 60; and 3*b*, equal 5 × 3, or 15, &c.

Note, where quantities are repreſented by letters, the product of the quantities may be expreſſed, without the ſign between them, by bringing them together, as in a word, thus, ab *is the ſame, as* a × b, *&c.*

÷ *By*, or the ſign of *diviſion*; and, when placed between any two quantities, ſignifies, that the former quantity muſt be *divided* by the latter; thus, *a* ÷ *b*, or (*a divided* by *b*), ſignifies the *quotient* ariſing from *dividing a*, by *b*; or, to uſe the numbers before taken for theſe quantities, *a* ÷ *b*, or (*a divided* by *b*), will expreſs the ſame, as 15 ÷ 5, or 3; that is, the *quotient*,

after dividing 15 by 5, &c. of any other.

Note, the diviſion of two quantities, can alſo be expreſſed, without the ſign between them, by putting the quantity or number you divide by, under the quantity or number to be divided; thus, $\frac{a}{b}$ *or* $\frac{15}{5}$, *is the ſame as* $a \div b$, *or* $15 \div 5$, *viz.* 3, *&c.*

$=$ *Equal*, is the ſign of *equality*, or *equation;* it ſignifies, when placed between two or more quantities, that, the quantity or quantities on the one ſide of it, will be equal to thoſe on the other ſide; thus, $c = a + b$, or $a + b = c$, where, ſuppoſe that a and b, equal the numbers before mentioned, then, the expreſſion $a + b = c$, will be, that $15 + 5 = c$; wherefore, c, which before had no value aſſigned to it, now gets a value; for, from having the quantities on one ſide of the ſign of equality known, the quantity repreſented by the letter c, on the other ſide, is therefore alſo known, it being equal to $a + b$, which, in this caſe, is 20; for, $15 + 5$, $= 20$, &c. of any other.

All ſuch expreſſions, as $a+b=c$, *&c. are called* equations, *becauſe, the quantities on one*

one ſide, are ſaid to be equal *to thoſe on the other; and, in geometrical arguments, ſuch expreſſions are extremely neceſſary; for, by uſing them, together with the* above *ſigns, or characters, inſtead of the words they repreſent, the demonſtration becomes ſhorter, and leſs confuſed; therefore, the reader muſt make himſelf well acquainted with them, and all other ſubſequent ſigns* * *he ſhall occaſionally meet with, if he be not ſo already.*

Q. E. F. *Quod erat faciendum*, which was to be done.

Q. E. D. *Quod erat demonſtrandum*, which was to be demonſtrated.

e. g. Exempli gratia, for example ſake.

(141) Figures, thus placed, refer to the *number* of the definition, problem, &c. where the authority is given for proceeding, in that particular manner, as directed, in that *definition*, *problem*, &c. to which the reference is made.

* There are, beſides the above, a few other characters to be explained, antecedent to the demonſtrative part, which, not being uſeful in this place, and to prevent burthening the readers memory, are omitted.

PART II.

PRACTICAL GEOMETRY.

POSTULATES.

137. I. That from any given point to another, a right-line may be drawn.

That is, a rule being laid from the point A, to the point B, (fig. 60) the line A B may be drawn with the point of a pen, or fine pencil; *this may be done alſo on the ground*, by ſtraining a line from A to B, having previouſly placed ſmall ſtakes, or pickets,* in the points A and B.

Hence, the definition of a right-line, is evidently proved to be the flowing of a point.

138. II. That a given right-line, may be continued ſtill further.

That is, the finite† line A B, (fig. 61) may be produced to D, or further, at pleaſure, by laying a rule (as in poſtulate 1) from A to B; *this may alſo be done on the ground*, by placing ſtakes or pickets at A and B, thus, ſtanding behind A, and looking towards B, let another ſtake, or, as many more as you pleaſe, be

* Pickets, are ſmall wooden pins about 18 inches or two feet long, generally uſed by engineers in drawing lines, &c. in the field, they are driven into the ground by a wooden mallet at proper diſtances, according to the figure deſigned to be executed; whereby they ſerve as directions to the workmen under them.

† A finite line, is that line which hath its terms given or known, whereby its length is determined.

placed

placed in ſuch a manner, that they may all appear to be in the ſame line, or behind B; then, lines ſtrained from one ſtake to the other, will continue the line as far as you chooſe.

139. III. That from any given point, as center, and with any given line, as radius, a circle, or an arch of a circle, may be deſcribed.

Thus, extend or open your compaſs, from the point A to the point B, (fig. 62) the given diſtance or radius; with this opening in your compaſs, remove one foot to C; then, by moving, or revolving the other foot round, the point C all the time kept faſt, this revolution will produce the circle; or, if you pleaſe, only a part, or an arch of it; and, if you draw the line C D, from the center to the circumference, it will be equal to the given radius A B. (*See rem. to def.* 14).

140. IV. That any right-line, or diſtance, may be taken, and transfered from one place to another.

For, in the laſt poſtulate, the line A B, was taken in the compaſs and transfered to the circle.

141. V. That one figure may be taken, and laid upon, or applied to, another, or conceived ſo to be done, whether it is really ſo or not.

142. VI. That when any one quantity is compared with, and found equal to, two others, reſpectively, either of theſe two quantities, may be taken and managed,

managed, the ſame as the quantity, with which it was found to be equal.

The reader, it is expected, will, before he proceeds, aſſent to the poſſibility of the above poſtulates, or, there muſt be an end to his purſuits; for, it will be impoſſible to conſtruct a ſingle figure in geometry, without they are firſt granted; as, *they* are principally the materials with which he muſt work; for, in every ſcience, mathematical, or otherwiſe, it is neceſſary ſome data ſhould be given, in order to proceed with certainty therein; and, the more ſimple, and plain that data is rendered, the more readily it will be aſſented to, and underſtood; conſequently, having leſs room for diſputation, he will proceed in the ſcience with ſatisfaction.

The young ſtudent is particularly recommended, to draw every figure as he proceeds, obſerving carefully, what points, lines, angles, &c. &c. are given him; which, he muſt always draw ſomething ſtronger, or more remarkable than the operative lines, which are generally dotted, or drawn much finer to diſtinguiſh them from each other. But, in marking down, or obſerving what things are given, he need not draw them exactly, as they are placed in the figure before him; but, only obſerve, that the rules laid down for him in each problem, &c. muſt be carefully attended to, let him draw his firſt line how he will.

e, g In the ſixth problem it is required, *to make an angle equal to a given angle, at any given point in a right-line.*

In this problem it is not determined in what poſition this line is to be drawn, on which the angle is to be made; beſides, the point, at which the angle is to be conſtructed, may be taken at either end of the line, or at any point in it; therefore, the angle may be compleated at either extremity, or at either ſide of the line; for, it is required only by the propoſition, to make

make the angle equal to the given one, which being done, is all that is requiſite.

In general, let him be careful, having firſt drawn his given line, or lines, to obſerve the directions laid down in the operative part of the problem, and proceed ſtep by ſtep, drawing every line, angle, &c. as directed; and, to be proper in every matter, let him preſerve his drawing inſtruments clean from dirt, always wiping his drawing pen in particular, clean and dry, when he has done uſing it, to prevent ruſt, as nothing is more detrimental than the ruſt from ink: in uſing his inſtruments, he muſt be careful to draw his lines clean, and neat; and, when only a blank or dotted line is to be drawn, let him lean very light on the inſtrument he uſes * for that purpoſe, to prevent cutting or ſcratching the paper, as nothing is more ſhameful than to ſee all the geometrical lines, neceſſary for the finiſhing any plan, &c. ſcratched or cut in the paper; on the contrary, nothing can be more creditable or pleaſing, than to ſee a clean, neat plan, from the draftſman.

Note, all dotted lines are beſt done, by uſing a common pen, (having firſt drawn a blank line, as a direction,) inſtead of the dotting pen in your caſe; for, unleſs the inſtrument is well made, the ink will clog in the wheel, and blot the line.

* For ſtraight or right-lines, uſe your pencil, or, only the ivory top of it, and for circular lines, the point of your compaſs, or the pencil point, which is in your caſe for that purpoſe.

PRAC-

PRACTICAL GEOMETRY.

PROB. I. 2 EUC. LIB. 1.*

143. From a given point, to draw a line equal to a given line.

Let the given point be C, *and the given line* A B. fig. 1.

METHOD 1. Extend or open your compaſs, from one extremity of the given line, as A, to the other extremity, B.

2. Keep that opening in your compaſs, and remove one foot to the point C; and on either ſide of C, as may beſt ſuit your purpoſe, ſweep the arch *a b*. (139).

3. Lay a rule from C, to any point in this arch, as D, and draw the line AD, which, will be equal to the given line. (137). *Q. E. F.*

On conſtruction.

The uſe of this problem is very evident, in drawing any plan on paper; for, by it a line of any given length can be taken from a plane ſcale, and transfered with convenience to the plan.

PROB. II. 10 EUC. LIB. 1.

144. To biſect, or to divide a line, into two equal parts.

Let the given line be A B. fig. 2.

* The numbers of the problems in *Euclid*, refered to, are the ſame as in *Whiſton*, which, being more generally uſed in the *College*, is therefore choſen.

METHOD

METHOD 1. From each extremity of the given line, A and B as centers, and with any radius *greater* than half A B, deſcribe two arches, cutting each other in C and D. (139).

2. Lay a rule from theſe points, where the circles cut each other, viz. in the ſections C and D, which will cut the given line in E, the point required. *Q. E. F.*

Def. 13. *and theorem* 1 & 3.

On the ground.

Strain a line from A to B, and double the line, by bringing the two ends A and B together, bring the ends thus doubled to A or B; a ſtake, (or picket) put in the ground between A and B, in the loop of the cord, pulling it tight, will be the middle required.

Note, the arch of a circle may be divided nearly in the ſame manner; for, let the chord of the arch be drawn, the ſame line C D, *that biſects the chord* A B, *will alſo biſect the arch in f.*

PROB. III. SCHO. 10 EUC. LIB. 6.

145. To triſect, or divide into three equal parts, any given right-line.

Let the given line be A B. fig. 3.

METHOD 1. With any convenient radius *greater* than half A B, make the two ſections *a* and *b*, (139) and draw the lines *a* A, and *a* B.

2. Biſect *a* A, and *a* B, in *c* and *d*. (144).

3. Draw the lines *c b*, and *d b*, and they will divide the line A B, as was required.

Theo. 38.

For another method of triſecting a line, ſee remark to prob. 9.

PROB.

PROB. IV. 11 EUC. LIB. 1.

146. To raiſe a perpendicular, from a given point in a right-line.

CASE I. *Let the given line be* AB, *and the given point* C, *at or* near the middle *of the line.* fig. 4. No. 1.

METHOD 1. On each ſide of the given point C, lay off any convenient, but *equal* diſtances *c b*, and *c d*. (140).

2. On theſe points, *b* and *d*, as centers, with any diſtance *greater* than *b* C, as radius, ſweep two ſmall portions of circles, cutting each other in D. (139).

3. From the ſection at D, draw a right-line to the point C, (137) and this will be the perpendicular required.

Theo. 3.

On the ground.

Provide a rod, or pole of 12 feet long,* which, let be divided into feet, and each foot into ten parts; with this rod meaſure from C, to *b* or *d*, 3 feet; then take a ſmall line, or cord of 9 feet long, in which, at the diſtance of 4 feet, make a knot; then, holding one end of the line at C, and the other at *b*, or *d*, let ſome other perſon take the knot of the cord in his hand, and draw it tight, and there drive a picket in the ground at D; ſo much of this cord as lies between C, and the picket at D, will be the perpendicular required.

* The great length of this rod may be obviated, by having it made in two joints, each of ſix feet long, and made to join like a meaſurer's rod; and, if the diviſions on it be numbered in the ſame manner, it will be found very convenient; as, it may be extended to anſwer any diſtance, above ſix feet, within the extent of it.

CASE

CASE II. *Let the given point* C, *be* at *or* near the end, *of the given line* AB, *from whence to make a right-angle.*

fig. 4. No. 2.

METHOD 1. Put one foot of your compaſs in C, and extend the other to any convenient point, over the given line, as *d*, but, let it be nearly in the middle between the given line, and where the perpendicular is to be drawn; on this point as center, and with the radius *d* C, deſcribe an arch greater than a ſemicircle, (139) which, let paſs through the given point C, and cut the given line in *e*.

2. Draw a line through *e* and *d*, and produce it until it cuts the circle in D. (137).

3. Draw the line C D, which will be the perpendicular required.

Cor. 3. to Theo. 28.

On the ground.

Lay your rod to the given point C, and, on the given line at the end of three feet, ſtick a picket; then, take the ſmall line or cord before uſed, and bring one end to C, and the other to the picket; which, let be drawn tight, and a ſecond picket put at the knot of the cord; now, if a line be ſtrained, or drawn from C to this ſecond picket, it will be the perpendicular required.

The uſe of perpendiculars, and right-angles, are ſo evident, that it is almoſt needleſs to point them out, particularly to builders, and other mechanicks; for the execution of their works not only depends entirely upon them, but they give to their work beauty, ſtrength, and convenience, &c.

PROB-

PROB. V. 12 EUC. LIB. 1.

148. To let fall a perpendicular to a given line, from a given point, *not* in the right-line.

CASE I. *Let the given point be* A, *nearly* over the middle *of the given line* BC. fig. 5. No. 1.

METHOD 1. On the given point A, as center, ſweep an arch with any convenient radius *greater* than the diſtance between the point and the line, which, let cut the given line BC, in *b* and *d*. (139).

2. Biſect the line *b d* in *e*, (144) or, take A *d* or A *b*, as radius; and, from the points *b* and *d*, make the ſection D. (139).

3. Draw the line A D, and ſo much of it as A *e*, which falls from A upon the given line, will be the perpendicular required. *Q. E. F.*

Note, when the rule is laid from A *to* D, *in order to draw the line* A D, *no more of the line need be drawn than* A *e, as no more of it is required.*

Theo. 3.

On the ground, when the diſtance is ſhort.

Proceed in the ſame manner as above, only uſe your 12 feet rod, inſtead of your compaſs; thus, take the diſtance between the given point A, and any point as *b*, in the given line, let the ſame diſtance be laid from A to *d*; lay the rod from *b* to *d*, and ſtick a ſtake or picket, at half the diſtance in *e*, then, ſtraining a cord, or lay the rod from A to *e*, and it will be the perpendicular required.

When

When the point A *is at a great diſtance.*

Let a cord be ſtretched from the point A, to any convenient point in the given line, as *b*, where, make a mark; keep this cord faſt at A, and, drawing it tight, let the end at *b* be removed to *d*, where, make another mark, then, at half the diſtance *e*, between *b* and *d*, ſtick a picket, and a line drawn from A to *e*, will be the perpendicular required.

149. CASE II. *Let the given point be* A, *and nearly* over the end *of the given line* BC. fig. 5. No. 2.

METHOD 1. From the given point A, to any convenient point, as *b*, in the given line, draw the line A *b*. (137).

2. Biſect the line A *b* in *d*. (144).

3. On *d*, as center, and with the radius *d* A, ſweep an arch, which will cut BC in D. (139).

4. Draw the line AD, which will be the perpendicular required.

Cor. 3. *to Theo.* 28.

On the ground.

Lay your 12 feet rod from the given point A, to any point in the given line; for inſtance, in *b*, ſtick a picket at the end of 6 feet, in *d*, then make the diſtance from *d* to D, equal 6 feet, and a line drawn from A to D, will be the perpendicular required; but, *ſuppoſing your* 12 *feet rod too ſhort*, take a cord of any convenient length, which double in the middle, and there make a knot, then ſtrain this cord, from the given point, to any point in the given line, as *b*, ſtick a picket by the knot in the cord, at *d*, then, keeping the middle of the cord faſt at *d*, remove the end of the cord from A to the given line, and it will give the point D;

D; from which, ſtrain a line to the given point, and it will be the perpendicular required.

PROB. VI. 23 EUC. LIB. 1.

150. To make an angle, equal to a given angle, at any given point in a right-line.

Let the given angle be D, *and the given point* A, *at the extremity of the given line* A B. fig. 6.

METHOD 1. From *d*, the point of the given angle, as center, with any radius ſweep the arch *a b*. (139).

2. From the given point A, in the given line, as center, with the ſame radius, ſweep the arch *c e*. (139).

3. Take the interval *a b*, (or the diſtance between the ſides of the given angle) and transfer it, from *c* to *f* in the arch *c e*, on the given line. (140).

4. Through the point *f*, draw the line A *f*, (137) and it will form the angle B A *f*, equal to the given angle D. *Q. E. F.*

Theo. 3.

Note, it might here be taught, how to meaſure an angle, but that will not be done, until a line of cords is firſt conſtructed, which will be ſhewn in prob. 31, *after which it will be better underſtood.*

PROB.

PROP. VII. 9 EUC. LIB. 1.

151. To biſect a given right-lined angle.

Let the given angle be A B C. fig. 7.

METHOD 1. From the point B of the given angle, with any *convenient* radius, ſweep the arch *a b*; this will mark the equal intervals, B *a* and B *b*, on the lines of the angle. (139).

2. On *a*, and *b*, as centers, with any interval *greater* than half *a b*, ſweep two arches, cutting each other in D. (139).

3. Draw the line D B, from the ſection D, to the angular point at B, and it will divide the angle. *Q. E. F.* (137).

Theo. 3.

On the ground.

Take, in the lines of the given angle, any equal intervals, as B *a* and B *b*, meaſure half the diſtance *a b* in *e*, (144) a line ſtrained from B through *e*, will divide the angle, as was required.

By this problem carpenters, joiners, &c. eaſily find what they call a mitre, *whether it be a right, acute, or an obtuſe angle, which is always half the angle, be it internal or external; and, without it be properly found, the mouldings will not fit exact, therefore, they are very careful in this problem.*

PROB. VIII. COR. 13. to 32 EUC. LIB. 1.

152. To triſect, (or divide into three equal parts) any given right-angle.

Let

Let the given right-angle be A B C.

fig. 8.

METHOD 1. From the angular point B, with any radius, ſweep the arch *b e*. (139).

2. Transfer the radius from *b* to *d*, and alſo from *e* to *c*, making the ſections *d* and *c*; if lines be drawn through theſe points *d* and *c*, they will divide the angle, as was required. (140).

On conſtruction, and cor. 7, *to theo.* 9.

PROB. IX. 31 EUC. LIB. 1.

153. To draw a line parallel to a given right-line, which ſhall paſs through a given point, or be at a given diſtance.

Let the given line be A B, *the given point* C, *and the given diſtance*, D.

fig. 9.

METHOD 1. *Suppoſing the point* C *given*, with one foot of your compaſs in C, as center, ſweep an arch, ſo as *juſt to touch* the given line in the point *n*, but, *by no means to cut it*. (153).

2. With the ſame interval, as radius, and from any other point in the given line as *e*, for a center, ſweep a ſmall portion of an arch over the given line. (139).

3. From the given point C, draw the line C E, ſo, as *juſt to touch this arch*, but *not*, on any account, *to cut it*, and this line C E, will be the parallel required.

Again, ſuppoſe the diſtance D *was given*, take that diſtance as a radius, and from any convenient points as *e*, and *n*, in the given line, ſweep ſmall portions of arches (139) over the given line, as in the ſecond method above, if a line C E be drawn, ſo, as *juſt to touch*, but, *not to cut* theſe arches, it will be the parallel required.

Theo. 8.

Note,

Note, the line AB, (prob. 3) *may be triſected otherwiſe, thus, having biſected a* A *in c* (fig. 3) *and drawn c b; from a, draw a line parallel to c b, and it will, with c b, divide it, as was required.*

On the ground.

Set the given diſtance perpendicular from any two points in the given line, and a line drawn through the extremities of theſe perpendiculars, will be the parallel required.

There are other methods uſed for drawing a line parallel to another, through a given point; (See 31 Euc. Lib. 1.) *but, they are omitted, as the above method is much ſhorter.*

PROB. X. 1 EUC. LIB. 1.

154. Upon a given line, to make an equilateral triangle.

Let the given line be AB. fig. 10. No. 1.

METHOD 1. From the given points A and B, and with the radius of the given line AB, deſcribe two arches cutting each other in C. (139).

2. From the point C, draw the lines C A and C B, (137) theſe lines will compleat the triangle required. *Q. E. F.*

Def. 3. *and Ax.* 1.

Nearly in the ſame manner, may any iſoſceles triangle, as A D B, fig. 10, No. 2, be conſtructed, the equal ſides of which, ſhall be of any given length, as C; for, from the extremities of the given line A and B, as centers, let two arches be ſwept with the

radius C (the given length) theſe will interſect in D, then draw the lines A D and B D, and the triangle will be compleated. *Q. E. F.*

PROB. XI. 22 EUC. LIB. 1.

155. To make a right-lined triangle, whoſe ſides muſt be reſpectively equal to three given lines, or, to the ſides of any other given triangle. fig. 11.

Let the three given lines, be the ſides of the triangle A B C.

METHOD 1. Draw the line D E equal A B (143) one of the ſides of the given triangle.

2. With the line A C, another of the given ſides, as radius, and on D, as center, ſweep the arch *a b*. (139).

3. In ſame manner from E, with the radius B C, ſweep another arch *d c*, cutting the arch *a b* in G. (139).

4. Draw the lines D G and E G, which will compleat the triangle required. *Q. E. F.*

On conſtruction.

Note, the proceſs would be the ſame, were the three lines of the triangle A B C, *given ſeparately. But, when three lines are given, any two of them muſt be greater than the third, or the problem will be impoſſible.* (See rem. on def. 42).

PROB.

PROB. XII. 46 EUC. LIB. 1.

156. To make a ſquare, on a given line.

Let the given line be AB. fig. 12.

METHOD 1. Upon the extremity B, of the given line, erect a perpendicular BC (146) which, make equal to the given line AB.

2. On the points A and C, as centers, with the radius of the given line AB, ſweep two arches interſecting in D. (139).

3. Draw the lines AD, and CD (137) which, will compleat the ſquare. *Q. E. F.*

Theo. 3, 8. *Cor.* 2, *to Theo.* 5, *and Def.* 52.

On the ground.

On A and B erect two perpendiculars, AD and BC, each equal the given line AB, a line drawn from D to C, will compleat the ſquare. *Q. E. F.*

PROB. XIII.

157. To make a rectangular parallelogram, on a given line, and of a given breadth.

Let the given line be A B, *and the given breadth be* C. fig. 13.

METHOD 1. Upon the extremity B, of the given line A B, raiſe a perpendicular (146 *Caſe* II) which, equal the given breadth C. (143).

2. On the point D, as center, and with the radius A B, ſweep an arch over the extremity A, of the given line. (139).

3. On the point A, as center, and with the interval C, ſweep another arch, cutting the former in E. (139).

4. Draw the lines D E and A E, which will compleat the parallelogram. *Q. E. F.*

Theo. 3, 8. *Cor.* 2. *Theo.* 5. *Def.* 50 *and* 51.

The performance of this on the ground is the ſame as the ſquare, which, to avoid repetition, is omitted.

The ſquare and rectangle, or parallelogram, is of great uſe in mechanicks, for moſt regular figures are rectangles, and, in the menſuration of moſt figures, their area is reduced to a ſtandard meaſure, by the ſquare or the rectangle, but more generally the ſquare.

Prob. XIV.

158. To make a rhombus on a given line, which ſhall contain a given angle.

Let the given line be A B, *and the given angle a.* fig. 14.

Method 1. On one extremity A, of the given line, make an angle (150) equal to the given angle *a*, with the line A D; which produce, until it is equal to the given line A D. (138).

2. On the points B and D, as centers, with the intervals of the given line A B, ſweep two arches cutting in C. (139).

3. Draw

3. Draw the lines BC, and DC, which will compleat the rombus required. *Q. E. F.*

Theo. 5, 8, *and Def.* 53.

Prob. XV.

159. To make a rhomboides on a given line, which ſhall contain a given angle, and of a given breadth.

Let the given line be AB, *and the given breadth* H, *and the given angle a.* fig. 15.

Method 1. On one extremity of the given line, as A, raiſe the perpendicular AD (146 *Caſe* II.) equal to the given breadth H.

2. Through the point D of this perpendicular, draw the *indefinite line*,* GF, of any convenient length, parallel to AB. (153).

3. Make at the point B, an angle equal to the given angle *a* (150) and produce its ſide BC, until it meets the parallel line GF in C. (138).

4. Take the interval AB, and transfer it from C to E, in the parallel GF. (140).

5. Draw the lines CE, and AE (137) which will compleat the rhomboides required.

Theo. 3, 8, *and Def.* 54.

Hence, a rectangle may be made equal to a rhombus or rhomboides, and the contrary; for, if on the baſe of the figure, as on AB (fig. 14 and 15) a rectangle be conſtructed, whoſe perpendicular altitude,

* Such lines are called *indefinite*, becauſe, their length is not given or determined, but, may be taken of any length at pleaſure.

ſhall

ſhall be equal BE or AD, it will be the rectangle required; again, ſuppoſe the rectangle, AF (fig. 15) was given, to make a parallelogram equal to it, containing a given angle; make the angle ABC upon one extremity of the baſe of the rectangle, equal to the given angle *a*, and conſtruct it ſo, as to lie between the ſame parallel lines with the rectangle, *and the parallelograms* AC *and* AF, *will be equal, (per. theo.*15).

Hence, *(and per rem. to def.* 52.) the area of a parallelogram is proved to be the product of the baſe into the perpendicular altitude, and the contrary.

PROB. XVI. 1 EUC. LIB. 3.

160. To find the center of a circle.

Let the given circle be EDFC. fig. 16.

METHOD 1. Draw in the circle any chord, as AB.

2. Biſect AB (144) with the line CD, which line CD, will be a diameter.

3. Biſect this diameter with the line EF, (144) and the point G, where AB and DC interſect, will be the center. *Q. E. F.*

Cor. 2, *to Theo.* 26.

PROB. XVII.

161. To divide the circumference of a circle into quadrants.

Let the given circle be ACBD.

fig. 17.

METHOD 1. Draw the diameter AB.

2. Biſect this diameter (144) with the diameter DC,

DC, and the extreme points A, C, B and D, of the diameters, will divide the circle as was required. *Q. E. F.*

On conſtruction.

Prob. XVIII.

162. To divide a circle into 12, 24, &c. equal parts.

Let the given circle be ABCD. fig. 18.

Method 1. Divide the circle into quadrants. (161).

2. Triſect each quadrant (152) in the points *a*, *b*, *c*, *d*, *e*, &c. which will divide the circle into 12 parts.

But, was it to be divided into 24 equal parts, biſect each twelfth part (*per note to* 144) and it will be done as you required: again, if theſe parts be biſected, the circle will be divided into 48 equal parts, &c.

On conſtruction.

Prob. XIX.

163. To divide the circumference of a circle into 8, 16, 32, &c. equal parts.

Let the given circle be ACBD. fig. 19.

Method 1. Divide it into quadrants. (161).

2. Biſect each quadrant (*per note to* 144) and you divide it into 8 equal parts.

3. Theſe

3. Theſe parts again biſected will divide it into 16 equal parts.

4. In the ſame manner biſect theſe parts, and you divide into 32 equal parts; and ſo on, by continual biſections, you may divide as far as you pleaſe in the ſame proportion. But,

As the continual biſection of arches will be very troubleſome and tedious, it may, in a great meaſure, be avoided, for, if you have carefully divided one ſemicircle as directed, you may draw lines from each diviſion of it, through the center; the extremities of theſe lines, meeting the oppoſite ſemicircle, will cut, or divide it in the ſame manner as the other; but, great care muſt be taken in dividing the firſt ſemicircle, for a trifling error in that one, will be transferred to the other, and conſequently, make a material difference in the diviſion of the circle required.

On conſtruction.

PROB. XX. 5 EUC. LIB. 4.

164. To deſcribe a circle through any three given points, *not in a right-line;* or, to circumſcribe a circle about a triangle; or, *(as Euclid has it in 25 Prob. 3 Book)* having part of an arch of a circle given, to perfect the whole circle.

Let the three given points be A, B, *and* C. fig. 20.

METHOD 1. Draw the lines A B, and B C. (137).

2. Biſect theſe lines with the lines *c d*, and *a b* (144) which lines let interſect each other in D.

3. From

3. From the point D, the interſections of theſe lines, as center, if a circle be deſcribed with the radius DA, the circumference will paſs through B and alſo C. *Q. E. F.*

Cor. 2, *to Theo.* 26.

Note, the proceſs is the ſame, was a triangle given; for, the angular points of the triangle, may be taken as the three given points; alſo, ſuppoſe an arch of a circle was given to be perfected, let three points be taken any where in the arch, and by proceeding as above, the center will be found, whereby the circle may be perfected.

Prob. XXI.

165. To make an equilateral triangle in a circle.

Let the given circle be A a, B b. fig. 21.

Method 1. From any point, as A, in the circumference, as center, ſweep an arch with the radius of the given circle (139) which arch, let cut the circumference in the points *a* and *b*.

2. Draw the chord *a b*, which will be the ſide of the triangle required: and the triangle may be compleated by laying this chord *a b*, from *a* to B, and drawing the lines *a* B, and B *b*.

Theo. 28.

Note, by biſecting the arches of a circle, including an equilateral triangle, as in c d, &c. and joining theſe points with the chord of each arch, you produce an inſcribed hexagon. But, was the hexagon alone to be inſcribed, (15 Euc. Lib. 4) *it is not neceſſary to make the triangle firſt; but, the method will be, to transfer the radius of a circle round the circumference, from any point, as* B, *to d, b,* A, *&c. and lines drawn to theſe points, will form the hexagon much ſhorter. And, if each arch of the hexagon be biſected, and*

and chords drawn to each arch, you thereby form a dodecagon, for, each arch being a ſixth part of the circumference, the half-arch, muſt be the twelfth part, &c.

PROB. XXII. 2 EUC. LIB. 4.

166. To inſcribe a triangle in a circle, which ſhall have equal angles to a given triangle.

Let the given triangle be a b c, and the given circle ABC. fig. 22.

METHOD 1. Draw the line DE, touching the circle in B, *which may be done, by laying a rule from the point* D, *ſo as juſt to touch, but not to cover any part of the circumference, and drawing the line by it.* (137).

2. Make the angle DBA, equal to the angle C, and the angle EBC, equal to the angle *a* (150) and continue the ſides until they meet the circumference in A and C.

3. Draw the line AC, and the triangle ABC will be the triangle required. *Q. E. F.*

Theo. 30.

PROB. XXIII. 3 EUC. LIB. 4.

167. To circumſcribe about a circle a triangle, which ſhall have equal angles, to a given triangle.

Let the given triangle be d e f, and the given circle BGC. fig. 23.

METHOD 1. Produce the ſide *e f*, of the given triangle, ſo as to make the external angles *n* and *o*. (138).

2. Make

2. Make at the center A of the circle, the angle BAC, equal to the angle *n*; and the angle GAC, equal to the angle *o*. (150).

3. Draw the lines DE, EF, and FD, perpendicular to AB, AC, and AG, and meeting each other in the points D, E, and F, which will compleat the triangle. Q. *E. F.*

Theo. 6, 27, 29. *Cor.* 3. *Theo.* 9, *and the Cor. to Theo.* 6.

PROB. XXIV. 6 EUC. LIB. 4.

168. To make a ſquare in a circle.

Let the given circle be ABCD. fig. 24.

METHOD 1. Divide the circle into quadrants. (161).

2. Draw the chords AB, BC, CD, and DA of each quadrant, and the ſquare will be compleated. Q. *E. F.*

On conſtruction, and Cor. 3, *to Theo.* 28.

Note, if each quadrant be biſected in a, b, c, and d, and chords to thoſe biſections drawn, the chords thus connected, will make an octagon in the circle; for the circumference is divided into eight equal parts; firſt, into four by the ſquare inſcribed, and ſecondly, into eight by biſecting each fourth part, for each quadrant is a fourth part of the circumference, conſequently, each half muſt be one-eight part.

PROB. XXV. 7 EUC. LIB. 4.

169. To circumſcribe a ſquare about a given circle.

Let

Let the given circle be ABCD. fig. 25.

METHOD 1. Divide the circle into quadrants, by the diameter AB and CD. (161).

2. Through the points D and C, draw the lines EF and GH, parallel to AB. (153).

3. Draw, in the ſame manner through A and B, EG and FH; parallel to DC, the meeting of theſe lines in the points E, F, G, and H, will compleat the ſquare. *Q. E. F.*

Theo. 14, 27, *and Cor.* 1. *Theo.* 32.

Note, the circumſcribed ſquare is always double the inſcribed ſquare, as may be proved from Theo. 16. *and a circle may be inſcribed in, or circumſcribed about a ſquare,* (8 and 9 Euc. Lib. 4.) *by taking the interſections of the diagonals as the center of the circle; and the radius of the inſcribed circle, will be the direct radius, and of the circumſcribed, the ſemidiagonal.*

PROB. XXVI.

170. To make a pentagon on a given line.

Let the given line be AC. fig. 26.

METHOD 1. Biſect the given line in E. (144).

2. Upon C, raiſe a perpendicular CD, which make equal to the given line. (146 *Caſe* II).

3. On E, as center, with the radius ED, ſweep an arch DF, meeting AC, produced in F. (139).

4. With the interval AF, and on A and C, the ends of the given line, ſweep two arches cutting each other in G. (139).

5. From the points A and G, and with the interval of the given line, ſweep two arches cutting each other in I, and in the ſame manner, with the ſame radius, ſweep arches from C and G cutting in K. (139).

6. Draw

6. Draw the lines AI, IG, GK, and KG, which will compleat the pentagon required.

On conſtruction.

To perform this on the ground, ſee after problem 32.

Pror. XXVII. 11 Euc. Lib. 4.

171. To make a regular pentagon in a circle.

Let the given circle be ADBG.

Method 1. Draw the diameter AB, on the middle of which, viz. the center C, raiſe the perpendicular CD. (147).

2. Biſect, or divide the radius CB in E. (144).

3. On E, as center, with the radius ED, deſcribe the arch EF. (139).

4. Draw the chord DF, which will be the ſide of the pentagon required.

5. From D, transfer the diſtance DF, to the points *a*, *b*, *c*, *d*, &c. in the circumference, and join them with the lines D*a*, *ab*, *bc*, &c. and the pentagon will be compleated. *Q. E. F.*

On conſtruction.

Note, if each arch of the circumferemce, ſubtending the ſides of the pentagon, be biſected and joined by chords, you will thereby form the decagon, for each arch is a fifth part of the circumference, therefore, the half of each will be a tenth part.

Prob. XXVIII.

172. To conſtruct or inſcribe in a circle, any regular polygon whatever.

Let the given circle be ABC, *and the polygon to be inſcribed an eptagon.* fig. 28.

Method

METHOD 1. Draw the diameter AB, which divide into 7 equal parts by repeated trials *(the polygon having 7 ſides)*.

2. On the points A and B, with the diameter, as radius, ſweep two arches cutting each other in D. (139).

3. From the ſection D, and through the ſecond diviſion of the diameter, draw the line D *d*; and B *d* will be the ſide of the eptagon required.

4. From *d*, transfer the line B *d*, round the circumference to 1, 2, 3, &c. and the chords joining theſe points, will compleat the eptagon required. *Q. E. F.*

Note, in conſtructing any other regular polygon, by this rule obſerve, that the diameter muſt always be divided into as many equal parts, as the polygon has ſides; and always draw the line from the ſection D, *through the* ſecond *diviſion of the diameter, and where it meets the circumference, it will cut off the ſide required.*

But, the conſtruction of any regular polygon in a circle, can be much more expeditiouſly and better performed by the help of a *ſector* (an inſtrument which every caſe of inſtruments ſhould be furniſhed with) thus, look for the line of polygons on the ſector, which is marked with the word *polygons*; then open the ſector, until the diſtance from 6 to 6, on that line, will be equal to half the diameter, or the radius of the circle; *which diſtance, muſt be previouſly taken in your compaſs, in order to open your ſector*, which is generally performed in this manner; on the line of polygons, at the No. 6, on both ſides of the ſector, is fixed a braſs point; on one of theſe points, put one foot of your compaſs, which, as I before obſerved, muſt firſt be opened equal to the radius; then, open the other ſide of the ſector, until the other foot of the compaſs will fall on the braſs point in that ſide, when that is done, the ſector is ſaid to be adjuſted; then, as the polygon is to be an *eptagon*, you muſt remove one foot of your compaſs,

paſs, nearer the joint of the ſector, to the No. 7; and, keeping the ſector as before adjuſted, cloſe your compaſs ſo much, that the other foot ſhall, in ſame manner, fall on the No. 7 in the other ſide of the ſector; this opening of the compaſs, if transferred to the circumference of the circle, will go ſeven times round; and, if ſections be made in the circumference, as the compaſs is carried round, and chords drawn to theſe ſections, the eptagon will be conſtructed as was required. Laſtly, you may, for any other polygon, having firſt adjuſted the ſector to the radius of the circle, open the compaſs from 4 to 4, for a *ſquare* 5 to 5, for a *pentagon* 6 to 6, an *hexagon*, &c. &c. and ſections made as before, and chords drawn will make the polygon required.

PROB. XXIX.

173. To deſcribe a circle, which ſhall contain any regular polygon, the ſides of which ſhall be of any given length.

Let the given ſide be A B, *and the polygon to be conſtructed a pentagon.*

fig. 29.

METHOD 1. Deſcribe the circle *a*, *b*, *c*, *d*, *e*, with any radius at pleaſure (139) in which, by the laſt problem, let a ſimilar polygon to that required, be inſcribed, which, in the preſent caſe is a *pentagon*.

2. Draw the radii *c a* and *c b*, and produce any ſide of the polygon, as *a b* to *h*, ſo that *a h* ſhall be equal to A B, the given length.

3. Draw *h* B parallel to the radius C *a*, which produce until it meets the radius C *b*, produced in B (138) and the line C B, will be the radius of the circle, which will contain the pentagon required.

The

The above problem can, as well as the former, be performed very conveniently with the ſector; thus, take in your compaſs the diſtance AB, and open the ſector, until the diſtance will reach from 5 to 5, on the line of polygons *(becauſe the polygon to be conſtructed is a pentagon)* keep the ſector exact and fixed to this opening *, and remove your compaſs from 5 to 5, and take the diſtance from 6 to 6 on the ſame line, which diſtance, will be the radius of a circle, that will contain the polygon required. But, ſuppoſe the polygon to be conſtructed was an *eptagon*, *octagon*, &c. you muſt firſt open the ſector, ſo that the given diſtance AB, will reach from 7 to 7, 8 to 8, &c. and the diſtance from 6 to 6, will always be the radius.

Note, this problem will be found of great uſe in fortification, *for, ſuppoſe a regular polygon was to be fortified, the ſide of which ſhould be of any given length, which is generally the caſe; by this problem, a circle is drawn, which will contain the required polygon, whereby, the polygon can be better conſtructed, than without it.*

Prob. XXX. 17 Euc. Lib. 3.

174. To draw a tangent to a circle which ſhall paſs through a given point.

Case I. *Let the given point be* A, *and in the circumference of the circle.* fig. 30. No. 1.

Method 1. Draw from the center C, into the given point A, the radius CA. (137).

2. On the point A, of the radius CA, raiſe the perpendicular BA (146 *Caſe* II.) which produce to D, or

* Obſerve always to keep the opening of the ſector unaltered, when once it is adjuſted to any radius, figure, &c.

any

any other convenient point, and BD will be the tangent required.

On conſtruction, and Theo. 27.

175. CASE II. *Let the given point be* A, *but not in the circumference.* fig. 30. No. 2.

METHOD 1. From the center C to the given point A, draw the line CA, which will cut the circle in *e*.

2. Biſect AC in B, (144) and from B, as center, with the radius BA, deſcribe an arch which will cut the circle in D. (139).

3. Draw the line AE through the point of interſection D, (137) and AE will be the tangent required.

Theo. 27, *and Cor.* 3. *Theo.* 28.

PROB. XXXI.

176. To divide the circumference of a circle into degrees, and from thence, to make a line of chords.

Let half the given circle be ABD. fig. 31.

METHOD 1. Draw the diameter ACD, (137) and on C raiſe the perpendicular CB, (146 *Caſe* I) which will divide the ſemicircle into quadrants. *(See rem. to def.* 30).

2. Triſect each quadrant in *a*, *b*, *d* and *c*, (152) then by trials, triſect each third part of theſe quadrants, and the ſemicircle will be divided into 18 equal parts, each containing 10 degrees.

3. Biſect each of theſe equal parts of 10, and you will have diviſions of 5 degrees.

4. Theſe arches of 5 degrees, divided by trials into 5 equal parts, will produce ſingle degrees.

Now, if the circle be compleated, and lines drawn through the center from each of theſe diviſions, to the oppoſite ſemicircle, the whole circumference will be divided into 360 equal parts. But, *in order to make a line of chords*, having finiſhed the fourth ſtep,

5. Put one foot of your compaſs in A, and open the other from that point, to each diviſion of the ſemicircle, and ſweep the arches 10, 10; 20, 20; *a* 30, &c. to the diameter, and theſe arches will divide the diameter into a ſcale of chords; that is, the ſeveral diſtances on the diameter, viz. A 10, A 20, A 30, &c. will be the chords of the arches A 10, A 20, A 30, &c. wherefore, the diameter will be a ſcale of chords to the whole ſemicircle, or 180 degrees.

The uſe of this ſcale in geometry, is in conſtructing and meaſuring all kinds of plane right-lined angles, to perform which, obſerve the two following problems.

PROB. XXXII. SCHO. 23 EUC. LIB. 1.

177. To make an angle of any propoſed number of degrees, on a given line.

Let the given line be AB, *on which you are to make angles of* 40 *and* 120 *degrees.* fig. 32.

METHOD 1. Take from the above, or any other ſcale of chords, the chord of 60° *, by opening the compaſs from A, or the *braſs point*, to 60, on the ſcale.

* This chord is always taken and uſed in conſtructing or meaſuring angles, becauſe, the chord of 60 degrees (AC), the *right-ſine* of 90 degrees (CB), and *radius* (AC or CB), in any circle whatever, are always equal.

2. With

2. With this diſtance, as radius, and from one extremity, as center, ſweep an arch from the point C in the given line, large enough to contain the degrees of the given angle. (139).

3. Take with your compaſs from the ſcale of chords, the chord of the given number of degrees, in the ſame manner as you took the chord of 60 degrees in the firſt method, and transfer theſe diſtances from the point C, where the circle begins, to D and E in the circle; (140) and draw the lines AD and AE, which will form the angles BAD of 40, and BAE, of 120 degrees. *Q. E. F.*

But as moſt ſcales of chords extend no further than 90 degrees, you muſt, when the angle is to contain more than 90 degrees, *e. g.* of 120, as above, always take the chord of half the number of degrees from the ſcale, and lay it twice on the arch, from *b*, to *c* and *d*, and the whole arch *b d*, will contain the degrees required.

On the ground.

Let the given line be AB, *and the angles as before.* fig. 32.

Take your 12 feet rod, and with it, from the extremity A of the given line, where the angle is to be conſtructed, meaſure off on the given line AC, equal 8.5 feet, or eight feet and an half; by ſtriking a picket in the ground at 8 feet and an half in C, on A raiſe the perpendicular AF (146 *Caſe* II) equal to 8.5 feet, or AC, then, if theſe diſtances and the perpendicular be taken right, your 12 feet rod will reach from C to F, obſerving to lay the *end* of your rod, or 12 feet at F, and the *beginning* at C; your rod now lying in the poſition FC, you may conſtruct the firſt angle, viz. of 40 degrees; thus, look in the following table in the column marked *degrees*, for 40, the number of degrees which the angle required is to contain, and oppoſite thereto, in the column under *diſtance*, you will find 5.5, that is, 5 feet 5 tenth-parts, or 5 feet and an half; at that diſtance from C, ſtick a picket in D cloſe

by the edge of the rod*, then, if a cord be ſtrained from A through D, cloſe by the ſide of the picket, it will make the angle required.

Again, to make the angle of 120 degrees, proceed, as before, to find the point F, which will (being in the perpendicular AF) form a right-angle, or an angle of ninety degrees; then, produce your given line from A towards H, where, let the *end* of your 12 feet rod, or number 12 fall; the *beginning* being firſt laid at F, your rod being now in the poſition FH, then, from the following table, and oppoſite to 30 degrees, *(that is, the number of degrees above* 90, *which your angle is to contain)* you will find 4.4, that is, 4 feet 4 tenth-parts, at which diſtance, from F in your rod, ſtick a picket in G, cloſe by the edge as before; and a line or cord, ſtrained from A through G, will make the angle FAG equal 30, which, added to the right-angle CAF before found, will make the angle GAC of 120 degrees, as was required.

Note, when the angle is obtuſe, as in the laſt example, it may be done much ſhorter, by producing your given line beyond the point A, *(on which you are to make the angle)* towards H; and, on AH, if you make the angle HAE equal to its ſupplement, it will, at the ſame time, make the obtuſe angle CAE, which was required.

* For this purpoſe in particular, it will be neceſſary to have the pickets made ſquare pins, by which means, in ſticking the picket by the edge of the rod, the flat ſide can lye cloſe to the rod, and the ſharp edge or angle be brought exactly to the diviſion, which could not be done with any degree of certainty if the pickets were round.

A TABLE,

A
TABLE,

By which, with the help of a 12 feet rod*, an angle of any number of degrees can be conſtructed or meaſured †.

	Diſt.	Cho.		Diſt.	Cho.		Diſt.	Cho.
deg.	f. 10p.	f. 10p.	deg.	f. 10p.	f. 10p.	deg.	f. 10p.	f. 10p.
1	0.2	0.1½	31	4.5	4.5½	61	7.7½	8.6½
2	0.4¾	0.3	2	4.6¾	4.7	2	7.8½	8.8
3	0.6	0.4½	3	4.7¼	4.8½	3	7.9¾	8.9¾
4	0.7½	0.6	4	4.8½	5.0	4	8.0¾	9.0½
5	9.9½	0.7½	35	4.9½	5.1½	65	8.2¼	9.1¾
6	1.1½	0.9	36	5.1	5.3	66	8.3¼	9.3
7	1.3¼	1.0½	7	5.1¾	5.4¼	7	8.4¼	9.4
8	1.4½	1.2	8	5.2¾	5.5½	8	8.5½	9.5¼
9	1.6½	1.3½	9	5.3¼	5.6¾	9	8.6¾	9.6½
10	1.8	1.5	40	5.5	5.8¼	70	8.8¼	9.7¾
11	1.9½	1.6½	41	5.5¾	5.9½	71	8.9½	9.9
2	2.1¼	1.8	2	5.7	6.1¼	2	9.0¾	10.0¼
3	2.2¾	1.9½	3	5.8	6.2½	3	9.2¼	10.1½
4	2.4¼	2.1	4	5.9¾	6.4	4	9.3¾	10.2¾
15	2.5½	2.2½	45	6.0	6.5	75	9.5	10.4
16	2.6¾	2.4	46	6.1	6.6½	76	9.6¾	10.5
7	2.8¼	2.5½	7	6.2¾	6.7½	7	9.7¾	10.6
8	2.9½	2.7	8	6.3	6.9	8	9.8½	10.7¼
9	3.1	2.8¼	9	6.4¼	7.0½	9	10.1¼	10.8½
20	3 2½	2.9¾	50	6.5½	7.2	80	10.2½	10.9½
21	3.3½	3.1	51	6.6½	7.3½	81	10.4	11.0½
2	3.4½	3.2½	2	6.7½	7.4¾	2	10.5½	11.1¾
3	3.5¾	3.4	3	6.8¾	7.6	3	10.7¼	11.3
4	3.7	3.5½	4	6.9½	7.7¼	4	10.8½	11.4
25	3.8¾	3.7	55	7.1	7.8¾	85	11.0½	11.5
26	3.9½	3.8½	56	7.2	8.0	86	11.2½	11.6
7	4.0½	3.9¾	7	7.3	8.1½	7	11.4½	11.7
8	4.1¾	4.1¼	8	7.4	8.2¾	8	11.6	11.8
9	4.2¾	4.2½	9	7.5	8.4	9	11.8	11.9
30	4.4	4.4	60	7.6¾	8.5	90	12.0	12.0

* If the length of this rod ſhould be inconvenient in meaſuring the chord of any confined angle, as, *an acute angle contained between two walls*, you may ſtrain a line, or take a ſix-feet rod divided like the 12. In ſuch a caſe if you uſe the numbers in the table ſimply as they are, *without conſidering them as feet*, they will anſwer equally as well; and indeed a ſix-feet rod *divided in this manner*, may anſwer in every inſtance as well as the 12, only, that the 12 feet *being a much larger ſcale* muſt be more exact; or, if a rod *ſo divided* cannot be conveniently had, you may, *by dividing a rule of one foot into tenths and quarter-tenths*, uſe any rod, for, *by applying this ſingle foot to any foot on the rod occaſionally*, it will anſwer the purpoſe.

† In *conſtructing* any angle, uſe the column under Diſtance; and in *meaſuring* any angle, the column under Chords.

In

In the above table, the ſeveral diſtances to be marked by the edge of the rod, in conſtructing or meaſuring any angle, is given in feet, and tenth-parts of a foot; the uſe of it (if not ſufficiently underſtood already, from conſtructing the foregoing problem) is as follows; after adjuſting the rod on the ground, as directed in the problem, look for the number of degrees the angle is to contain, in the column marked *degrees* in the table, and oppoſite to each degree, is the diſtance in feet, tenth-parts, and quarter tenth-parts, to be taken, or marked off by the edge of the rod; thus, was the angle to be made of 15 degrees, look for 15 degrees, as directed in the column marked *degrees*, and oppoſite thereto, you will find 2.5¼, which is two feet five-tenths and half a tenth, to be taken by the edge of the rod. Again, ſuppoſe the angle was to be 64 degrees, look for 64 degrees in the table, as before, and oppoſite thereto, you will find 8.0¾, that is, eight feet and three quarters of a tenth, to be marked by the edge of the rod, from C towards F, as before.

But, if it be required to make an angle equal to any number of degrees, and minutes, as ſuppoſe of 64° 30′ * (the table not being calculated to minutes) the angle will be made ſufficiently exact for any military or other purpoſe, if, to the number annexed to 64 degrees, you add half the difference between that and 65 degrees, viz. three-fourths of a tenth *(for the whole difference being .1¼ tenth, or ſix-fourths, the half therefore muſt be three-fourths)* you will make the angle required; thus, inſtead of 8.0¾ feet you muſt mark off 8.1½ feet, that is, eight feet one-tenth, and half a tenth; therefore, by always adding to the degrees of the angle, the ſame part of the diſtance or difference as the given minutes are of a degree, the angle will be compleated; and, in order to be certain what part of this difference is to be added, obſerve, that for 15 minutes you muſt add one-fourth, for 30 minutes (as was the caſe in the above example) one-half, and for 45 minutes three-fourths. But, ſhould the number of odd minutes in the angle not exactly anſwer to the above numbers, always uſe ſuch of them

* Theſe marks °, ′, ſignifies degrees and minutes.

as

as ſhall be neareſt thereto, and your angle will be made exact enough for any purpoſe on the ground.

Having now ſhewn how to make an angle of any number of degrees, it becomes neceſſary (according to promiſe after prob. 26) to ſhew how

To conſtruct any regular polygon on the ground.

Let the polygon to be conſtructed be the pentagon AIGKC. fig. 26.

Firſt, Draw A C, one ſide of the pentagon, of any given or convenient length.

Secondly, On A and C, the extreme points of the line AC, make angles equal to 108 degrees, which are the number of degrees contained in the angle of the pentagon, *(for, the degrees in the angles of any polygon, taken together, are always equal to twice as many right-angles as the polygon hath ſides, abating four, ſee theo.* 11) that is, in this caſe (the polygon being a regular one) to the fifth part of 6 right-angles, or 540 degrees.

Thirdly, Produce the ſides of theſe angles to K and I, ſo that AI, and AK, be each, equal to AC; and on I and K make angles, as before, equal to 108 degrees; and the meeting of the ſides of theſe laſt angles in G, will compleat the pentagon required, which when done, you may prove to be rightly conſtructed, by finding the center (*per* 164) and, if the diſtance from the center to each angular point be equal, the polygon is right. Or thus, produce AC, to any convenient diſtance F, and make the angle FCK equal the fifth-part of 360 degrees, (the polygon having five ſides) which will be equal the angle at the center of the polygon, viz. 72 degrees, *(ſee cor. theo.* 12) do the ſame at the point A, whereby you make the angles KCA, and CAI, equal 108°, as above, &c. for the reſt proceed as before.

PROB.

Prob. XXXIII. Scho. 23 Euc. Lib. 1.

To meaſure any right-lined angle.

178 Case I. *Let the given angle be* ABD, *leſs than* 90 *degrees.* fig. 33.

Method 1. Take from the ſcale of chords, the chord of 60°, *(as directed in the laſt problem)* and from B, the point of the given angle, as center, ſweep an arch between the ſides of the given angle, which will cut them in *a* and *b*. (139).

2. Take the interval of the arch *a b* in your compaſs, *which will be the chord of that arch*, and tranſfer that diſtance to the line of chords, thus, put one foot of your compaſs on the braſs point in the beginning of the line, and, whatever degree in the line the other foot ſhall fall upon, that will be the meaſure or quantity of degrees in the angle required.

179 Case II. *Let the given angle be* ABE, *more than* 90 *degrees.* fig. 33.

Method 1. Sweep the arch *b c d*, as in the firſt ſtep in the former caſe, and, on the arch, lay off from *b* to *c*, the cord of 60°; which, as was before obſerved, will be the radius of the arch.

2. Take the interval *c d*, and meaſure it, as in the 2d ſtep of the former caſe, and, to the degrees it is found to contain, add the degrees of the arch *b c*, viz. 60°, and the ſum will be the content of the angle; thus, was the diſtance *c d*, found to be 59 degrees, the meaſure or content of the angle would be 59° + 60°, viz. 119°. *Q. E. F.*

This may alſo be done on the ſector, thus, open the ſector to any convenient radius, but not too large *(for obſerve, every opening of the ſector is a different radius)* and open your compaſs from the braſs point at 60, on one leg,

leg, to 60 on the other, in the line chords (marked cho. on each leg); with this interval ſweep an arch from the angular point B, as before, which, will mark on the legs of the angle the points *b* and *a*; now take the interval *b a*, *(which is the chord of the arch)* in your compaſs, and transfer it to the ſector, thus, lay one point of your compaſs on one leg of the ſector, in the line of chords, which, move lightly backwards and forwards in that line, until the other leg of the compaſs will cut, or fall on the ſame degree, on the oppoſite leg of the ſector, and whatever degree that is, it will be the meaſure of the angle. In the ſame manner may the obtuſe angle ABE be meaſured, having firſt laid the radius, from *b* to C, the arch *c d* being meaſured, as before, and added to *b c*, or 60°, the ſum will give its content, &c.

But, in uſing the ſector, be careful to apply the compaſs to that parallel, in the line which is neareſt the opening or inner edge of the ſector, and in which line you will always find the braſs point placed.

Note, in meaſuring of angles more than 90°, and the arch very large, or the angle very obtuſe, *lay off the radius, or chord of* 60°, *as often as you can on the arch that meaſures the angle; which, obſerve can never be more than twice* (for three times the radius will meaſure 180°, or a ſemicircle) *then, if any part of the arch be extended beyond the diſtance of double the radius, the remaining arch being meaſured, and added to twice radius, or* 120 *degrees, the ſum will be the content; or, you may firſt meaſure off a right-angle, by laying the chord of* 90°, *on the arch, and, to* 90°, *add the remaining part of the arch, and the content will be the ſame as before. But, the* former method being ſhorteſt, it is recommended.

On the ground.

Let the angle to be meaſured be the *acute angle* ABD, (fig 33) *firſt* lay off on each leg of the angle from B, to *a* and *b*, eight feet and half; then meaſure with your rod, the diſtance between *b* and *a* which, ſuppoſe, for example, was found to be three feet four-tenths, look for that diſtance in your table under the column of chords, and the degrees anſwerable thereto, being 23, is the meaſure of the angle. Proceed in the ſame manner

manner for all angles leſs than 90°. But, ſuppoſe the *oqtuſe angle* ABE, (fig 33) was to be meaſured, *firſt*, lay off on each leg of the angle, as before *a b*, and *a d*, each equal eight feet and half; *ſecondly*, raiſe the perpendicular B *e*, (146 *Caſe* II) which, make alſo equal eight feet and half, and the diſtance *b e*, will be 90°, or a right angle; *thirdly*, meaſure the acute angle E *b d*, as before, and add its content to 90°. and the ſum will be the quantity, or meaſure of the angle ABE; or, you may *meaſure its ſupplement* by producing the line AB, which will determine the angle ABE. *(ſee note, page* 84). *Q. E. F.*

But, ſuppoſe the diviſion on the rod, cut by the legs of the angle, cannot be found exactly in the table of chords, *e. g.* five feet two-tenths; in that caſe, obſerve the degrees anſwering to the *chords* next *leſs*, and next *greater* than five feet two-tenths; and note the difference between *(which difference will always equal one degree, the table being calculated only to degrees)* and in this example will be, the difference between 5 feet .1½ tenth, and 5 feet three-tenths, viz. .1½ tenth, that is, the angle is more than 35°, and leſs than 36; which are the degrees anſwerable to the *chord* next *leſs* and next *greater*. Now, obſerve what part of this difference, added to the leſſer *chord*, will make 5 feet two-tenths, the *chord* found on the rod; which, in this example will be half a tenth, that is one-third part of it, *(for* 5.1½ + 0.0½ *will equal* 5.2) therefore the angle is to be one-third part of a degree *more* than 35°, which is 35°, 20′; conſequently, when theſe degrees are added to 90, the ſum will be the quantity or meaſure of the angle BAE, viz. 125° 20′, &c. of any other.

Note, if at any time the ſides of the angle to be meaſured ſhould not be long enough, they muſt be produced to a ſufficient length.

Prob. XXXIV. Cor. 47, Euc. Lib. 1.

180. To draw the *right-ſine*, *co-ſine*; *tangent*, *co-tangent*; *ſecant*, and *co-ſecant*, of

of an arch, or, to an angle of any number of degrees. (*See from def.* 77 *to* 82)

Let the given arch be ED, *the meaſure of the angle* BAD. fig. 34.

METHOD 1. Draw the radius AB, (if not already drawn) and on its extremity B, raiſe the perpendicular BC, which, produce until it will meet AC, (drawn from the center of the arch, through D, the other extremity of it) in C.

2. From D (the extreme point of the arch cut by AC) to the radius BA, let fall the perpendicular DE, and DE will be the *right-ſine*, BC the *tangent*, and AC the *ſecant* of the angle BAD, or of the arch BD. *Q. E. F.*

3. To find the *co-ſine*, *co-ſecant*, and *co-tangent*; draw (from the center A) the radius AF, perpendicular to AB, (146 *Caſe* II.) and from D, to AF, let fall the perpendicular DH, (149) alſo from F, raiſe the line FG perpendicular to AF, (146 *Caſe* II.) and DH will be the *co-ſine*, FG the *co-tangent*, and AG the *co-ſecant*. *Q. E. F.*

But, was the *ſine*, *tangent*, &c. of an arch *greater than* 90° required, find the *ſine*, *tangent*, &c. of the ſupplement to the arch, and it will be done; for ED is the *right-ſine* of the arch IFD, as well as of the arch DB, &c. or, the *ſines* of 60° and 120°, of 40° and 140°, &c. are equal.

Note, when the given arch is the meaſure of a right-angle, or contains 90 *degrees, the right-ſine will always be the radius of the arch; but, the* tangent *and* ſecant *of* 90 *degrees, cannot be determined, for, was* AF *(the ſine of* 90°*) produced, it would never meet the* tangent HC, *but would continue to be parallel to it; therefore, the* ſecant *and* tangent *of* 90°, *are infinite, conſequently not determinable.*

PROB.

Prob. XXXV.

181. To meaſure the angles of any triangle, which together, always make 180 degrees.

Let the given triangle be DEF. fig. 35.

Method 1. Meaſure each angle ſeparately, *(per laſt prob.)*

2. Add the quantity or content of each angle together, and their ſum, if it be equal to 180 degrees, will prove they were meaſured right; if not, they muſt be meaſured over again until they do.

Theo. 9.

It is recommended to be very attentive to this problem, until you are very expert in meaſuring the angles, for, the making or meaſuring of angles accurately is of great conſequence, particularly where any calculation is depending.

Prob. XXXVI.

182. From the line of chords, to inſcribe any regular polygon in a circle.

Let the polygon to be inſcribed, be a decagon. fig. 36.

Method 1. Sweep a circle with the radius of 60°, from the line of chords.

2. Divide 360 (the number of degrees in the circumference) by 10, the number of ſides in the polygon, and the quotient, viz. 36°, if taken from the line

line of chords, will, when transfered to the circumference of the circle, go ten times round, viz. in the points *a*, *b*, *c*, *d*, &c.

3. Draw chords to thoſe arches, and you thereby compleat the polygon. (137).

But, as few plane ſcales have more than one line of chords on them, if you want at any time a larger or a ſmaller circle than your ſcale will allow, you muſt uſe the ſector; for, by opening it, until the radius you want will reach from 60 on one leg, to 60 on the other in the line of chords, you thereby adjuſt the ſector to that radius, and any number of degrees can be taken from the ſector, as well as from the plane ſcale; thus, put one foot of your compaſs to the number of degrees on one ſide of the ſector (obſerving to keep the ſector fixed) and open or ſhut the compaſs, until the other foot will fall on the ſame number of degrees, on the other leg, and that opening in the compaſs, will be the diſtance or chord of the number of degrees required.

Prob. XXXVII. 12 Euc. Lib. 4.

183. About any circle to draw a regular polygon.

Let the given polygon to be drawn, be a pentagon and the given circle be a b c d e. fig. 37.

Method 1. Inſcribe in the circle a ſimilar polygon to that which is to be circumſcribed, viz. the pentagon *a b c d e*, (*per* 171, 172, *or* 182).

2. Parallel to the ſides of the inſcribed polygon, draw the *tangent* lines AB, BC, CD, &c. (174) and the

the meeting of theſe *tangent* lines in the points A, B, C, &c. will make the poylgon required. *Q. E. F.*

Cor. to Theo. 32. *Theo.* 3. *Cor.* 6, *to Theo.* 28. *Cor.* 2, *to Theo.* 27, *and Theo.* 4.

PROB. XXXVIII. 13 EUC. LIB. 4.

184. To inſcribe a circle in any regular polygon.

Let the given polygon be the hexagon a b c d e f. fig. 38.

METHOD 1. Find the center C, of the polygon, which will be the point of interſection of two lines, biſecting any two of its adjacent ſides. (164).

2. Draw the perpendicular CA, (146 *Caſe* I) and with it, as radius, deſcribe a circle, which will be the circle required; *or, find the direct radius* (ſee def. 92) *and it will be the radius of the inſcribed circle.*

Theo. 2, 4, 27, *and Cor.* 2, *to Theo.* 5.

PROB. XXXIX. 14 EUC. LIB. 4.

185. About any regular polygon to circumſcribe a circle.

Let the given polygon be the hexagon a b c d e f. fig. 39.

METHOD 1. Find the center C, of the polygon (*per method* 1, *prob.* 38).

2. From

2. From the center C, draw a line to any angle of the polygon as C *c*, (137) and it will be the radius of the circle required.

PROB. XL. 33 EUC. LIB. 3.

186. On a given line to deſcribe a ſegment of a circle, which ſhall contain a given angle.

Let the given line be AB, *and the given angle c.* fig. 40.

METHOD 1. Below, or under the given line A B, and on one of its extremities, as A, make the angle B A F, equal to the given angle C. (150).

2. Biſect the given line in E (144) and raiſe the perpendicular E D. (146 *Caſe* I).

3. From the point A, draw A G perpendicular to A F (146 *Caſe* II) which will cut F D in G.

4. From G, as center, with the radius G A, ſweep the arch A C B (139) which, with the chord A B, will be the ſegment required.

Theo. 30.

PROB. XLI. 34 EUC. LIB. 3.

187. From a given circle to cut off a ſegment, which ſhall contain a given angle.

Let the given circle be ADBC, *and the given angle b.* fig. 41.

METHOD

METHOD 1. To any point in the circumference, as A, draw the tangent line E F. (174).

2. At the point of contact A, make the angle F A C equal to the given angle *b* (150) by drawing the line A C, which line A C, will cut off the required segment A B C. *Q. E. F.*

Theo. 30.

PROB. XLII. COR. 42 EUC. LIB. 1.

188. To make a triangle equal to a given quadrilateral figure.

Let the given quadrilateral figure be A B C D. fig. 42.

METHOD 1. Draw the diagonal line B D (137) and parallel to it, draw the line C E (153) meeting A D produced in E.

2. Draw the line B E, and the triangle A B E will be the triangle required. *Q. E. F.*

Theo. 14, *and Cor. to Theo.* 15.

PROB. XLIII.

189. To make a triangle equal to a five-sided figure.

Let the given figure be A B C D E. fig. 43.

METHOD 1. Draw from any of its angles, as C, the lines C A and C E. (137).

2. From

2. From the points B and D, draw the lines B F and D G, parallel to C A and C E, (153) and meeting A E, produced in F and G.

3. Draw the lines C F and C G, (137) and the triangle F C G will be that which was required.

Theo. 14, *and Cor. to Theo.* 15.

Prob. XLIV.

To make a parallelogram equal to a given triangle.

190. Case I. *Let the given triangle be* A B C, *and the parallelogram to be conſtructed a rectangle.* fig. 44. No. 1.

Method 1. Biſect the baſe A C in D. (144).

2. Through B draw the indefinite line E H, parallel to A C. (153).

3. Raiſe the perpendiculars A E and D F *(per* 146, *Caſe* I *and* II*)* which will make the rectangle A E F D, equal to the triangle A B C. *Q. E. F.*

Part 3, *to Theo.* 14, *and Cor. to Theo.* 15.

191. Case II. *Let the parallelogram required be conſtructed ſo, as to contain a given angle.* (42 Euc. Lib. 1).

Let the given angle be a. fig. 44. No. 1.

Method 1. Proceed as in the firſt and ſecond methods above.

2. Make the angle G A C, equal to the given angle *a*, (150) and let the line A G, meet the line E H, in G.

3. From the point D, draw the line D H, parallel to A G, which alſo let meet the parallel E H in H, and it will compleat the parallelogram A H, that which was required.

Theo. 14.

192. CASE III. *Suppoſe the parallelogram was required to be made on a given line, and to contain a given angle.*

44 Euc. Lib. 1.

Let the given line be A B, *and the given angle o.* fig. 44. No. 2.

METHOD 1. Make the line A *e*, equal half the baſe of the given triangle, (143) and on it *(per Caſe* II *of this)* make the parallelogram A *f* equal to the given triangle A B C, having the angle *f e* A, equal to the given angle *o*.

2. Through B, draw the line B *h*, parallel to A *g*, meeting *f g* produced in *h*. (153).

3. Draw the diagonal *h* A, which produce until it meets F *e* produced in *i*. (137, 138).

4. Draw *i* D parallel to A B, and produce the lines *g* A and *h* B, until they meet the line *i* D, in C and D; which will make the parallelogram A D, equal to the triangle A B C. *Q. E. F.*

Cor. to Theo. 14.

PROB. XLV.

193. To make a rectangle equal to a given trapezium.

Let the given trapezium be A B C D. fig. 45.

METHOD

METHOD 1. Draw the diagonal DB, and from A and C, let fall the perpendiculars A*a* and C*c*. (148).

2. Biſect theſe perpendiculars in *d* and *b* (144) and through *d* and *b* draw *gf* and *he* parallel to the diagonal DB. (153).

3. Through D and B draw *gh* and *fe*, perpendicular to DB (149) which will make the rectangle *efgh* equal to the trapezium ABCD. *Q. E. F.*

PROB. XLVI. 18 EUC. LIB. 6.

194. On a given line to make a rectilineal figure, ſimilar to a given one.

CASE I. *Let the given line be* AD, *and the given figure the triangle* EGF.

fig. 46. No. 1.

METHOD 1. On A, one extremity of the given line, make the angle BAD equal the angle GEF. (150).

2. On D, the other extremity of the given line, make the angle ADB equal the angle EFG, and the triangle ADB will be ſimilar to the triangle EFG. *Q. E. F.*

On conſtruction, and Def. 75.

195. CASE II. *Let the given line be* AD, *and the given figure the quadrilateral* EFGH. Or, let the given figure have as many ſides as you pleaſe.

fig. 46. No. 2.

METHOD 1. Divide the given figure into the triangles EFG and FGH, by the diagonal line GF. (137).

2. Make the triangle ABD ſimilar to the triangle EFG (*per Caſe* I) and on BD make the triangle BCD, ſimilar to the remaining triangle of your given

figure CHF, and the quadrilateral figure ABCD will be ſimilar to EGHF. *Q. E. F.*

On conſtruction. Theo. 9, *and Def.* 75.

Note, was the given figure to contain more than four ſides, proceed, as in this caſe, to divide it into triangles; and, by conſtructing ſimilar triangles in the ſame manner, as above, a ſimilar figure will be produced; becauſe all right-lined figures are compoſed of, or may be reduced to, right-lined triangles. Therefore, &c.

Or you may proceed more readily thus; ſuppoſe the figure to be the pentagon ABCDE, and the given line B*f*, (*fig.* 46, *No.* 3); *Firſt*, from any angle of the given figure, as B, draw, of any convenient length, indefinite fine lines with a black lead pencil, &c. through each angle; and alſo, produce indefinitely the ſides BA and BC of the angle B. *Secondly*, take your given line B*f*, and tranſfer it from B towards A, on the line BA, as to *f* (140). *Thirdly*, from *f* draw *f k*, parallel to AE (153) which let meet BE, formerly produced in *k*; and, in the ſame manner, from *k* and *i*, draw *k i* and *i h*, parallel to ED and DC, and the pentagon B*f k i h* will be ſimilar to that which was given (*per theo.* 43). But obſerve, if the given line B*f* was *greater* than BA, the point *f* would fall beyond A, and the polygon conſtructed would be *greater*, and fall entirely on the outſide of the given polygon. On the contrary, if B*f* was *leſs*, the polygon would therefore be *leſs*, and lie entirely within the given polygon. Hence, right-lined figures of any form may be enlarged or diminiſhed in any given proportion; for, was the line B*f* twice as long as BA, the increaſed or enlarged polygon would be four times as large as the given one; if three times as long, nine times as large, &c. (*per theo.* 43). And again, if B*f* was but one-half, or one-fourth of BA, then the conſtructed or diminiſhed polygon would be only one-fourth, or one-eighth part of the given one; that is, the given one would be four times, or eight times *greater* than the conſtructed. (*Per cor.* 3, *to theo.* 43).

43). *But in order to determine that line, on which you may enlarge or diminiſh any polygon in a given proportion, ſee prob.* 56, *Caſe* I. *and* II.

PROB. XLVII. 11 EUC. LIB. 6.

196. Two right-lines being given to find a third proportional.

Let the given lines be A *and* B. fig. 47.

METHOD 1. Draw two indefinite right-lines, making any angle, as *a*.

2. Transfer the given line A, from *a* to *b*; and B, from *a* to *d*, and alſo from *a* to *c*. (140).

3. Draw *b d*, and parallel to it draw *c e* (153) and *a e* will be the third proportional.

That is, *a b* : *a d* : : *a c* : *a e*. (*Per theo.* 38).
or A : B : : B : *a e*. *Q. E. F.*

PROB. XLVIII. 12 EUC. LIB. 6.

197. Three right-lines being given to find a fourth proportional.

Let the given lines be A, B *and* C. fig. 48.

METHOD 1. Draw two indefinite right-lines, making any angle, as *a*.

2. On theſe lines take *a b* equal A, *a c* equal B, and *a d* equal C. (140).

3. Draw *b c*, and parallel to it draw *d e* (153) and *a e* will be the fourth proportional.

That is, A : B : : C : *a e*. (*Per theo.* 38). *Q. E. F.*

PROB.

PROB. XLIX. 13 EUC. LIB. 6.

198. Two right-lines being given to find a mean proportional.

Let the two given lines be A *and* B. fig. 49.

METHOD 1. Draw any indefinite right-line, in which take *a b* equal A, and *b c* equal B. (140).

2. Biſect the line *a c* in F (144) and on F, as center, with the radius F*a*, deſcribe the ſemicircle *a d c*. (139).

3. From *b (the point where the given lines meet)* raiſe the perpendicular *b d*, which will be the mean proportional required.

That is, a b : b d : : b d : b c. (Per Cor. 1, *to theo.* 41).
or A : *b d* : : *b d* : B. *Q. E. F.*

PROB. L.

199. The ſum of two extremes, and the mean proportional given, to find the two extremes.

Let A B *be the ſum of the two extremes, and the line* C *the given mean proportional between them.* fig. 50.

METHOD 1. Biſect A B in G (144) and on G, as center, ſweep the ſemicircle A E B. (139).

2. On B raiſe the perpendicular B D (146, *Caſe* II) which make equal to C, and draw D E parallel to A B (153) which will cut the circle in E.

3. Draw

3. Draw E F parallel to B D, which will cut A B in F, the point where the two extremes meet; ſo that A F and F B will be the two extremes required. *Q. E. F.*

On conſtruction, and Cor. 1. *Theo.* 41.

Prob. LI.

200. The difference between two extremes, and the mean proportional given, to determine the two extremes.

Let A B *be the difference of the two extremes, and* C *the mean proportional.* fig. 51.

Method 1. Draw an indefinite right-line, as F G, in any convenient part of which take B A, equal the given difference A B. (140).

2. Biſect B A in D (144) and on B raiſe the perpendicular B E (146) which make equal to C. (143).

3. Draw D E, and with it, as radius, and from D, as center, ſweep the ſemicircle F E G, and F B and B G will be the two extremes. *Q. E. F.*

On conſtruction, and Cor. 1. *Theo.* 41.

Prob. LII.

201. The difference between the diagonal of a ſquare and its ſide being given, to determine the ſide of the ſquare.

Let the given difference be G. fig. 53.

Method

METHOD 1. Draw any indefinite right-line, and from one extremity of which, take A B equal to the given difference G. (140).

2. On B raiſe the perpendicular B C (146, *Caſe* II) which make equal to A B (143) and draw A C, which produce towards D, or until C D is equal to C B, and A D will be the ſide of the ſquare required. *Q. E. F.*

On conſtruction, and Theo. 16.

PROB. LIII. 11 EUC. LIB. 2.

202. To cut a given line into extreme and mean proportion; *that is, to cut it in ſuch a manner, that the rectangle under the* whole line, *and* one part, *ſhall be equal to the* ſquare *of the* other part.

Let the given line be A B. fig. 53.

METHOD 1. Biſect the given line in C, (144) and on one of its extremities B, raiſe the perpendicular B D, (146, *Caſe* II) which, make equal to B C, and draw A D.

2. From the point D, as center, and with the radius D A, ſweep the arch A E, which, let meet D B produced in E.

3. Take the interval B E and transfer it on the line B A, from B to F, and F will be the point required.

That is, a *ſquare* made on the line B F, will be equal to a *rectangle* under the whole line A B, and the remaining part F A, for it will be A B : F B :: F B : F A, that is, the rectangle, &c.

Theo. 16, 21, *and Ax.* 3.

PROB.

PROB. LIV. 10 EUC. LIB. 6.

203. To divide a given right-line in the ſame manner, or in the ſame proportion, as another given line C is divided.

Let the given line to be divided be AB. fig. 54.

METHOD 1. From one end A, of the given line AB, draw the indefinite line A*a*, making any angle with it, on which lay off, from A to D, A 1, A 2, &c. equal the ſeveral diviſions, as given on the line C, and AD will be equal to C.

2. Draw the line DB, and parallel to it draw lines through the ſeveral diviſions of AD, (153) and theſe parallels will cut the given line AB, as was required.

Theo. 38.

PROB. LV. SCHO. 10. EUC. LIB. 6.

204. To divide a line into any number of equal parts.

Let the given line be AB; *to be divided*, e. g. *in ſix equal parts.* fig. 55.

METHOD 1. Draw an indefinite line AC, making any angle with the given line; and on AC lay off as many convenient, but equal diſtances A*b*, *b c*, &c. to *g*, as your line is to be divided into, viz. ſix.

2. Draw

2. Draw gB, and parallel to it draw, from the ſeveral diviſions on AC, the lines $f h$, $e i$, &c. (153) which will divide AB, as was required.

Theo. 38.

Note, if, when you have drawn AC, *as above, and laid the diviſions on it, you draw a line from* B *parallel to* AC, *making an alternate angle to* BAC, *and mark the ſame diſtances on it as on* AC, *then let lines be drawn ſo as to join thoſe diviſions on each parallel, and where they croſs the given line in the points h, i, k, &c. they will divide the line, as was required. But, in this caſe, the diſtances to be laid on the parallels need not be ſo many as the parts to be contained in the line, but always one leſs, which in the example above will be only five from* A *to f.*

PROB. LVI. COR. 3, to 20. EUC. LIB. 6.

To enlarge or diminiſh any *polygon, triangle, ſquare,* &c. in any given proportion.

205. CASE I. To enlarge any triangle, ſquare, &c.

Let the line DC *be the ſide of any* polygon, triangle, *&c. ſimilar to which I would make another three times as large.* fig. 56.

METHOD 1. Produce the given line from C unto B, (138) ſo that CB ſhall be three times CD.

2. Find the mean proportional CE, between DC and CB, (198) and CE will be one ſide of the polygon required. That is, if you conſtruct on CE a ſimilar polygon to that which was given on CD, (*per* 154, 170, 194, &c.) it will be three times as large. *Q. E. F.*

Cor. 3. *Theo.* 43.

But,

But, was the *polygon* to be conſtructed, to be 4, 5, &c. times as large, make CB, 4, 5, &c. times as long as the given line CD; or, was it required to make it $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, &c. times as large, in this caſe, let DC firſt be divided into quarters, halves, &c. and make CB = $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, &c. times as large as DC, and find the mean proportional, as before; on which, if you conſtruct a ſimilar polygon (154, 170, 194, &c.) it will, &c. Thus, *e. g.* was the polygon required to be $2\frac{1}{2}$ times as large, make CB = 5 times the one-half of CD, becauſe $2\frac{1}{2}$ = 5 halves; and if $2\frac{3}{4}$ make CB = 11 times the $\frac{1}{4}$ of CB, for $2\frac{3}{4}$ = eleven quarters or fourth-parts, &c.

206. Case II. To diminiſh any triangle, ſquare, polygon, &c.

Let the line BC, *be the ſide of any* triangle, ſquare, *&c. ſimilar to which I would have one made, one-third leſs.* fig. 56.

Method 1. Divide the given line BC into three equal parts, (145 or 204) and produce BC unto D, or until CD = $\frac{1}{3}$ of BC.

2. Find the mean proportional CE, between BC and CB, (198) and if on CE you conſtruct a ſimilar triangle, ſquare, polygon, &c. it will be = $\frac{1}{3}$ of that which was given upon BC. *Q. E. F.*

But, was it required to make the triangle, &c. $\frac{1}{4}$, $\frac{1}{5}$, &c. leſs than the *triangle*, &c. upon BC, divide BC into 4, 5, &c. equal parts, and make CD = $\frac{3}{4}$, $\frac{1}{5}$, &c. of BC, and find the mean proportional as before, on which, if you conſtruct a *triangle*, &c. it will be but $\frac{1}{4}$, $\frac{1}{5}$, &c. of the given one. *Q. E. F.*

Prob.

Prob. LVII.

To make a ſquare equal to any right-lined figure.

207. Case I. *Let the given figure be the* rectangle ABCD. fig. 57.

Method 1. Find the mean proportional D*e*, between its ſides A D and D C. (198).

2. On D*e* conſtruct a ſquare (156) and it will be equal to the rectangle. *Q. E. F.*

Theo. 16, 21, *and Ax.* 3.

208. Case II. *Let the given figure be the* triangle A B C. fig. 44.

Method 1. Make the rectangle A E F D, equal to the given triangle. (190).

2. Make *(per Caſe* I) a ſquare equal to the rectangle, and it will be that which was required.

209. Case III. *Let the given figure be a* rhombus, *or* rhomboides.

Method 1. Make a rectangle equal to either of them *(per rem. to* 159).

2. Make a ſquare, as before, equal to the rectangle, and, &c.

210. Case IV. *Let the given figure be a* polygon.

Method 1. Reduce it to a triangle. (189).

2. Make

2. Make a rectangle equal to the triangle (190).

3. Make a ſquare equal to the rectangle *(Caſe* I. *of this)* and, &c.

Prob. LVIII.

211. To make a plane ſcale, or a ſcale of equal parts, uſed for laying down or meaſuring *lineal diſtances*, or *right-lines*, uſually eſtimated in feet, yards, &c. *(See rem. to def.* 6). fig. 58.

Method 1. Draw three right-lines A, B and C, of any convenient length (ſufficient for your ſcale) parallel to each other (153) and at the ſame diſtance, as in the figure,

2. Draw an indefinite line, as E D, perpendicular to the line C (146, *Caſe* II) and in the line C take the equal intervals *c a*, *a b*, *b c*, &c. to the end of the ſcale, of any propoſed length, as an inch, one-half, or one-quarter of an inch, &c. which diſtance in the ſcale call the *principal diviſions*. (140).

3. On the points *c*, *a*, *b*, &c. draw lines perpendicular to, and acroſs the parallels A, B and C.

4. Take any point in the line E D, as D, and draw D F of any length perpendicular to E D, and on D F lay off ten equal intervals *(not too large, but all together greater than* C *a)* and draw the line F *a*, which, if produced to the line E D, would cut it in E.

5. Lay a rule from the point E to each diviſion in the line D F, and let fine lines be drawn to the line C *a*, and they will divide it into ten equal parts*, like F D; then, if from each diviſion of C *a* lines be drawn parallel to E D, acroſs the parallels B and C, the principal diviſion C *a* will be divided into ten equal diviſions, called *ſub-diviſions*.

* Hence there appears another method of dividing a line into any number of equal parts; for, if F D was divided into 4, 5, &c. parts, and lines drawn, as above, to E, they would cut *a c* in the ſame manner into 4, 5, &c. parts.

6. Let

6. Let each principal diviſion on the ſcale be numbered from the right hand towards the left, beginning at *a*, with 0, 1, 2, 3, &c. as in the figure, and the ſcale will be finiſhed.

Note, if your principal diviſions were made equal to one inch, half *an* inch, *&c. the ſcale would be nominated accordingly; that is, an* inch, *or* half-inch ſcale, *&c. and, as the uſe of theſe ſcales is for meaſuring and conſtructing of figures, whoſe ſides are required or given in feet, yards, &c. it will be neceſſary to ſhew on this ſcale,* firſt,

212. How to meaſure any given line or diſtance.

Take in your compaſs the interval of the given line or diſtance, and lay one foot upon any of the principal diviſions, ſo that the other may fall either upon 0, or among the ſub-diviſions; then, in the *former inſtance*, the number of principal diviſions, which may be conſidered as 1, 10, 100, &c. feet, yards, &c. will be the length required. But in the *latter*, you muſt, to the number of principal diviſions, add the number of ſub-diviſions on which the other foot ſhall fall, counting from 0 towards C, thus, if each principal diviſion was eſtimated one foot, yard, mile, &c. then each ſub-diviſion, being $\frac{1}{10}$, or a decimal of the principal, you muſt add ſo many tenths of a foot, yard, mile, &c. But, if each principal diviſion was ſuppoſed to be 10, 100, &c. miles, then would each ſub-diviſion be 1, 10, &c. miles; that is, the fourth ſub-diviſion would be 4, 40, &c. miles, always one-tenth of the principal. And *ſecondly*,

213. How to take off any lineal diſtance, *e. g.* 240 feet, &c.

You muſt firſt value the principal diviſions, according to the diſtance to be taken; that is, conſider them as 1, 10, 100, &c. feet, &c. which, in this caſe, muſt

muſt be ſuppoſed equal to 100 feet; then, if you put one foot of your compaſs on the ſecond principal diviſion, and extend the other until it will fall upon the fourth ſub-diviſion, the interval in the compaſs will be the extent required.

Again, was the diſtance to be taken off equal to 246 feet, &c. you muſt, in this caſe, after putting one foot of your compaſs in the ſecond principal diviſion, as before, extend the other until it will fall a little more than half way between the fourth and fifth ſub-diviſions; becauſe the diſtance is *more* than 240, and *leſs* than 250; that is, whatever part of 10 feet, &c. any odd number of feet, &c. in the given number is, the ſame part of any ſub-diviſion muſt be taken and added. But, when the diſtance to be taken is given in odd feet, &c. as in the laſt example, always uſe a diagonal ſcale, it being more exact; to conſtruct which, ſee the following problem:

Prob. LIX.

214. To make a diagonal ſcale of equal parts. fig. 59.

Method 1. Draw the line A B of any convenient length, ſufficient for the purpoſe of your ſcale, which divide into the primary and ſub-diviſions, as in laſt problem.

2. Parallel to, and below A B, draw ten lines at any ſmall but equal diſtances, as in the figure, each equal to A B (153) and from each principal diviſion let the perpendiculars B D, o C, &c. be drawn acroſs the parallels (146) whereby each parallel will be divided like A B.

3. Divide the principal diviſion C D of the lower parallel into ten equal parts, as you divided o B, and let oblique lines be drawn from the diviſions of o B to thoſe of C D, as in the figure, acroſs the parallels, and the points where they cut them will determine the

ſeveral

ſeveral diſtances between 10 and 100, 100 and 1000, &c.

4. Let each principal and ſub-diviſion, as alſo the parallels, be numbered, as in the figure, and the ſcale will be compleated.

The ſuperiour advantage of this ſcale to the former, will evidently appear from the uſe of it, for which ſee the two following problems: Firſt,

215. To meaſure any line or diſtance.

Proceed, as directed in 208, to meaſure the given line on the line A B; and, ſhould the foot of the compaſs which falls among the ſub-diviſions in o B, not exactly lie on any particular diviſion, as it generally happens, you muſt, in that caſe, obſerve what principal diviſion the other foot of your compaſs was placed on, and gently ſlide that foot down the perpendicular, from that diviſion acroſs the parallels, moving the other foot at the ſame time; let this motion be continued, until the foot on the ſub-diviſions ſhall fall upon ſome interſection amongſt the diagonal lines; and the parallel line, on which this interſection ſhall happen, will be the length required. Thus, ſuppoſe one foot was on the ſecond principal diviſion in the ſixth parallel, and the other upon the interſection of the fifth diagonal, with the ſame parallel in the point *a*; then the diſtance *b a*, being added to the value of two principal diviſions, will determine the diſtance; as ſuppoſe, one principal diviſion was 10, 100, 1000, &c. feet, yards, &c. then the diſtance would be 25.6, 256, 2560 feet, yards, &c. Again, *ſecondly*,

216. To take off any lineal diſtance, as 25.6, 256, 2560, &c. feet, &c.

In

In the firſt example, each principal diviſion muſt be 10; in the ſecond, 100; and in the third, 1000, &c. feet, yards, &c. then, in each caſe, open your compaſs from the ſecond principal diviſion on the ſixth parallel, to the 5th diagonal, and you have the interval required; always obſerving, to put the compaſs on that principal diviſion which anſwers to the higheſt figure of the given diſtance, and denominate each according to the value of that figure. But, to meaſure any diſtance

On the ground.

You muſt obſerve, that all lines or diſtances, the lengths of which are undetermined, muſt be meaſured by ſome determinate line, as, of a certain number of feet, yards, perches, chains, &c. Now, *ſuppoſing the diſtance very large*, a ſurveyor's chain of either two or four perches long is generally uſed; the former having 50, and the latter 100 links. One end of this chain is carried forward in the line to be meaſured, and the perſon who carries it ſticks a picket, or an iron pin, at the end of the chain, which is taken up by the perſon who carries the other end of it, who, before he picks up the pin, always takes particular care that the chain is drawn tight, and exactly in the direction of the line to be meaſured. When they arrive at the end of the line, the number of pins taken up, added to the number of odd links beyond the laſt pin, will be the number of chains and links in the line: conſequently, the length of one chain being known, the line to be meaſured is alſo known in chains, perches, &c.

Again, *when the diſtance is ſhort*, and great exactneſs required, uſe the 12 feet rod in the ſame manner as the chain was uſed, and you will have the length in feet, and decimals of a foot. Or, if you chooſe, you may uſe a rod or ſtaff of 10 links in length; and, if each link be divided into 10 parts, you will have the diſtance in links, and decimals of a link.

But, notwithſtanding the preceding method is very exact, yet, in military purpoſes, ſuch exactneſs not being abſolutely neceſſary; therefore other methods have been thought on, which have been found to anſwer ſufficiently exact, thus, *e. g.* To meaſure the diſtance of two objects, between which an encampment is to be formed, the officer, whoſe buſineſs it is to ſettle theſe matters, rides over the ground a horſe, whoſe gait and rate of going he is very well acquainted with, between the objects, and by the help of his watch, obſerves the time ſpent in going from one object to the other. Now, ſuppoſe his horſe would walk 110 yards in one minute, and that he was 100 minutes going from one object to the other, then would 100 × 110 = 11000 yards be the diſtance. Or, ſuppoſe in cantering his horſe, he would paſs over 250 yards in a minute; in that caſe, ſuppoſe he was 40 minutes going the diſtance, it would be 250 × 40 = 10000 yards. Therefore it can be determined ſufficiently exact, whether the diſtance will admit of the purpoſe or not.

Or diſtances may be meaſured by walking; that is, by the number of paces or ſteps taken in the line, which, if done with care, may come very near: but, firſt meaſure the length of a pace, by going backwards and forwards over a certain meaſured ſpace of ground, each time counting the number of paces, which, when done two or three times, you thereby can determine how many you take; then divide the length by the number of paces, and the quotient will be the length of one pace on an avarage: thus, *e.g.* ſuppoſe your pace to be 28 inches, multiply the number of paces in any required diſtance by 28, and the product will be the length required.

It is particularly recommended to young gentlemen intended for military purpoſes, to make themſelves expert at trials of this kind; and alſo, to conſtruct and meaſure, by the eye, as near as poſſible any kind of angle; raiſe or let fall a perpendicular from and to a line, and ſuch uſeful problems; exerciſes of this kind, will make them very expert when they come

come to be engineers, for by them they will be able to diſcover, at ſight, whether the buſineſs they are executing is right or not; therefore, they ſhould take convenient opportunities for this purpoſe, and have a rod, &c. always ready to determine whether they have been right or not.

Note, From the method of conſtructing this diagonal ſcale, a line may be very conveniently divided into any number of equal parts; if you draw any line, as B D, and at very ſmall but equal diſtances; alſo draw ſeveral parallel lines, perpendicular to B D (as in fig. 59) the number to be determined at pleaſure; which, when carefully done, you may divide any line, by laying it acroſs them from B, thus, *e. g.* to divide a line into ſix equal parts; take the interval of the given line, and put one foot of your compaſs in B, and lay the other on the ſixth parallel, and obſerve where it falls; from which, if you draw a line to B, the parallels will divide it (that is, your given line) into ſix equal parts, as was required.

Again, to divide a line into 12, &c. equal parts, proceed as above, only obſerve, that there muſt be as many parallel lines drawn as are neceſſary for the number of parts you would divide the line into; or, as the diviſion of a line in conſtructing of any plane ſcale (particularly thoſe for architecture) is neceſſary to be fully attended to, you may proceed, as directed in prob. 58, method 4, by dividing a line, as D F, carefully into 10, 12, &c. equal parts (204) and draw fine lines from each diviſion of D F to any point, as E, in a line perpendicular thereto, as E D, which call *oblique lines:* by theſe lines you may divide a line into equal parts, from a quarter of an inch, or leſs, to any length within the diſtance D F, thus, *e. g.* to divide a line of half an inch into 12 equal parts, parallel to and at half an inch diſtance from E D, draw a fine pencil line acroſs the twelfth oblique line, and where it cuts it, let fall a perpendicular to E D, which will be half an inch long, and the oblique lines croſſing it will divide it into 12 parts. Proceed in the ſame manner for any other diſtance.

Prob. LX.

217. To conſtruct a ſcale of feet and inches, generally uſed by maſons, carpenters, &c. in drawing their plans. fig. 60.

Method 1. Draw the lines A B and C D at any convenient diſtance, as in the figure, and on C D lay off your principal diviſions, as in the former ſcales, of one inch, half, or quarter of an inch, &c.

2. Parallel to, and under C D (as in ſecond method of prob. 59) draw ſix lines, and let the perpendiculars B F, o E, &c. be drawn from each principal diviſion acroſs theſe lines.

3. Biſect E F in 6, and draw the diagonal lines o 6, 6 D, which will make the ſub-diviſions on the parallels.

4. Number the principal and ſub-diviſions, as in the figure, and the ſcale will be compleated.

Note, The method of taking off, or meaſuring any diſtance on this ſcale, being the ſame as on the diagonal ſcale, (prob. 59) *it need not be again repeated; only obſerve, as the principal diviſions on this ſcale are always feet, the ſub-diviſions therefore cannot be decimals, but twelfth-parts of the principal diviſions; that is, any number of ſub-diviſions will be ſo many inches to be added, as before.*

Prob. LXI.

218. To make a ſcale of fathoms and feet, uſeful in fortification, drawing profiles, &c. fig. 61.

Method 1. Draw the parallel lines A and B *(diſtant as per figure)* of any convenient length, for your ſcale,

ſcale, and parallel to them draw three other lines, as in the former ſcales.

2. Lay off on B D your primary diviſions, as before, which, in this ſcale, will always be one fathom or ſix feet.

3. Biſect the primary diviſion E F, of the lower parallel, in 3, and draw the oblique lines o 3, 3 D, which, croſſing the parallels between D and F, will give the ſub-diviſions, or the feet in each fathom. Or you may draw ſix parallel lines, as in the laſt ſcale, for the ſix feet in the fathom, and draw one diagonal line from o to F, and the ſub-diviſions will be in that diagonal.

To take off any diſtance on this ſcale, *e. g.* Ten feet, or one fathom four feet.

Put one foot of your compaſs in the firſt primary diviſion and the ſecond parallel line (that being ſix out of the 10 feet) and open the other foot to the fourth ſub-diviſion, marked 4, on that parallel (for the remaining four feet) and the interval will be the ten feet required, &c. of any other.

Prob. LXII.

To meaſure the area of any right-lined figure.

219. Case I. *Let the given figure be the* ſquare A B C D. fig. 12.

Rule. Multiply the ſide B A by itſelf, and the product will be the area required; for, *as each ſide of a ſquare is equal* (per def. 53) *let the baſe* B A *be of what meaſure you pleaſe, the area muſt be the product of that line by itſelf.* See rem. to def. 52, and theo. 15.

220. Case

220. CASE II. *Let the given figure be the* rectangle ABDE. fig. 13.

RULE. Multiply the baſe AB by the ſide BD, or multiply the length by the breadth, and, in either caſe, the product will be the area. *Per rem. to def.* 52, *and theo.* 15.

221. CASE III. *Let the given figure be the* rhombus ABCD (fig. 14) *or* rhomboides ABCE. fig. 15.

RULE. Multiply the baſes AB by the perpendicular altitudes BE and AD, and the products will be the areas required. *Per theo.* 15.

222. CASE IV. *Let the given figure be the* triangle ACB. fig. 44.

RULE. Multiply the baſe AC by half the perpendicular altitude DF, or the perpendicular altitude DF by half the baſe AC, and, in either caſe, the product will be the area required. *Per theo.* 14, *and cor.* 1, *to theo.* 15.

223. CASE V. *Let the given figure be the* trapezium. fig. 62.

RULE. Divide the given trapezium into triangles, by the diagonal line AC, and find the area of each triangle *(per caſe* iv. *of this)* and the ſum of their areas will be the area required. *(Per theo.* 15, *and ax.* 9). Or thus, multiply the ſum of the perpendiculars in each triangle by the diagonal, and half the product will be the area required.

And if any irregular polygon, or any right-lined figure, be given to find its area, you muſt firſt divide the figure into *triangles* and *trapeziums*, by drawing diagonal lines; and if you find the areas of each *(per caſe* iv *and*

iv *and* v *of this)* the ſum of all the areas together will equal the area of the whole figure.

224. CASE VI. *Let the given figure be any* regular polygon.

RULE. Multiply half the perimeter *(def.* 87) by the direct radius *(def.* 92) and the product will be the area required: *for the area of any polygon will be equal to a rectangle, the baſe of which is equal to half its perimeter, and altitude the direct radius. This will be plain, on conſtructing the parallelogram, and dividing it into equal triangles with thoſe of the polygon.*

PROB. LXIII.

225. To meaſure the area of a circle. *(See def.* 93).

Before this can be done with propriety, it will be neceſſary to premiſe the following particulars:

1. As a circle contains the largeſt area under the leaſt limits, it is therefore the moſt extenſive of all figures *.

2. As a circle may eaſily be conceived to be a polygon of an infinite number of ſides *(ſee note to def.* 86) as of 1000, 10,000, &c. ſides; or if you will 12,600 ſides; that is, the number of minutes in 360 degrees:

* For, if a circle be depreſſed ever ſo little, it will become elliptical, or like an oval; that is, a *circular figure, longer one way than another:* in which caſe it will loſe of its dimenſions the more it is depreſſed, that is, the more excentric any oval or ellipſis is, having equal circumference or periphery with a circle, the leſs will be its area to that of the circle; for, depreſs it until it becomes flat, and it will loſe its area entirely: conſequently, in *all regular polygons*, the more the ſides are encreaſed, the nearer will the figure be to that of a circle, and contain the greater area; that is, polygons of a greater number of ſides contain a greater area than thoſe of fewer ſides, having equal perimeters.

which

which ſmall portion of the circumference will approach infinitely near to it, or the ſides of ſuch a polygon would nearly terminate in it; therefore, a circle may, without any ſenſible error, be conſidered as equal to a *right-angled triangle*, whoſe baſe is the circumference, and perpendicular the radius.

3. Since every *circumſcribed polygon* is *greater*, and every *inſcribed polygon leſs* than the circle; it follows, that the area of the former polygon muſt be *greater* than that of the latter; therefore, as the area of this triangle is *leſs* than the circumſcribed polygon, and *greater* than the inſcribed one, it will evidently be equal to the circle. Hence,

4. A *rectangle* under the *radius*, and *half* circumference, will be equal to the circle. A *rectangle* under the *radius*, and *whole* circumference, will be double the circle. And laſtly, a *rectangle* under the *whole* circumference, and the *diameter*, will be four times the circle. Hence, to find the area of a circle, the rule is,

226. *Multiply half the circumference by half the diameter*, or *the whole circumference by one-fourth of the diameter; and*, in either caſe, *the product will be the area.*

But, to find a right-line which will exactly equal the circumference of a circle, notwithſtanding much labour and time has been ſpent in various calculations, has not yet been diſcovered: hence the reaſon why the area of circles cannot, with certainty, be aſcertained; and, to make up for this defect, approximations have been uſed for finding the circumference, having the diameter given, and from thence, as near as poſſible, to arrive at the true area, as that of *Van Culen* of *Leyden*, where, he ſuppoſed the diameter to be 1.000000.000000.000000, &c. and the circumference 3.141592.653589.793238, &c. of which large numbers, no more need be uſed than the firſt five or ſix; that

that is, if the diameter be 1.00000, the circumference will be 3.14159. Now, notwithſtanding much time and labour was ſpent in calculating the above numbers, others have alſo been uſed, as 7:22; as 106:333; as 113:355; as 1702:5347; as 1815:5702, &c. &c. by any of which the circumference of any circle may be found, from having the diameter given, thus,

: 7 : 22
: 106 : 333 } :: any diameter : the circumference,
: 113 : 355
&c.

of which proportions the reader may uſe which he likes; but for his government let him obſerve that that of *Van Culen* is more exact; but for practice, that of 7:22, though not ſo true, is more generally uſed. However, as this is more a book of *geometry* than *figures*, let us proceed

227. To find the circumference of a circle, or an arch of it, geometrically.

To perform this, various methods have alſo been uſed, many of which, like the above calculations, come very near the truth: however, the Author *begs leave to introduce one of his own, which, on trial, will be found as exact as any to be met with. It is as follows:*

Let the given circle be A B C D.

fig. 63.

Draw the diameter A C, and at right-angles to it the diameter B D; lay the radius from D to E, and draw E B, which will cut the diameter A C in F; or make C E equal one-third of the quadrant B C, and draw B E. Or laſtly, draw the chord B G equal the radius, and B E perpendicular thereto, and, in either caſe, the line B E will cut the diameter in F, and A F will be very nearly equal to the arch of the quadrant of the circle. Conſequently, if A F be carefully meaſured on a good diagonal ſcale, and its length taken, four

four times the product will be extremely near the circumference, and from thence the area may be found.

Now, if the above method be tried, and compared with the area found by proportion, it will appear how near they are to each other.

EXAMPLE.

Let the radius of a circle AO (*fig.* 63) be 11.35, and taken from a good diagonal scale, like that in prob. 59, No. 214; on which let each principal division be considered as equal to 10 feet, &c. Now, if AF be found and measured on the same scale, it will be equal to 17.83: therefore, $17.83 \times 4 = 71.32$, the whole circumference, and from the foregoing proportions it will be equal to 71.31.

Thus,

106 : 333 :: 22.7 * : 71.31

333

25197 × 3

106 | 7559.1 | 71.31

139

331

130

24

&c.

Or thus,

113 : 355 :: 22.7 : 71.31

355

1135

7945

113 | 8058.5 | 71.31

148

355

160

47 &c.

And by the proportion of 7.22 the circumference will be equal to 71.34, more than, *viz.* the other proportions.

7 : 22 :: 22.7 : 71.34

22

7 | 4994

71.34

So that the area, by calculation from the circumference found in the above proportions, will be 404.$\frac{3}{4}$ feet nearly.

* *This* 22.7 = 11.35 × 2; *that is, to the diameter, or double the radius.*

EXAM-

EXAMPLE.

35.655 = half circumference,
11.35 = radius, or half diameter,

178275
4029015

404,68425 = area in feet, &c.

And by the diameter found from geometry, the area will be very nearly the ſame. As for

EXAMPLE.

2 | 71.32 = whole circumference,

35.66 = half ditto,
11.35 = radius, or half diameter,

17830
402958

404.7410 = area in feet, &c.

In all which, the number of ſquare feet in area is the ſame, *viz.* 404. But a difference appears only in the decimals, which by *geometry* is but .06 of a foot more than that found by *calculation*; and if the area was found from the circumference produced from the proportion of 7.22, it would be equal 404.85 feet, where the number of feet is ſtill the ſame, but the decimals greater than any other; and, in all of which, the decimals may, without error, be eſteemed three-fourths of a foot.

PROB. LXIV.

228. To meaſure a ſector of a circle. (*See def.* 24). fig. 64.

RULE.

Multiply the radius BA by the arch BDC, or the contrary, and half the product will be the area.

And

And in the performance of this, it will be neceſſary to ſhew how, in

Prob. LXV.

229. To find a right-line nearly equal the arch of a circle, or to any part of the circumference.

Let the given arch be CBA. fig. 65.

Divide the chord of the given arch CA into four equal parts, and lay one of thoſe parts on the arch from A to B, and draw a line from B to the third diviſion on the chord CA, as B 3, and double that line will be nearly equal the arch.

Prob. LXVI.

230. To meaſure any ſegment of a circle. *(See def.* 23).

Let the given ſegment be BDC. fig. 66.

Draw the radii BA, CA, and meaſure the ſector BDCA (228) and from the area of this ſector ſubtract the area of the triangle ABC, found *(per* 223) and the difference or remainder will be the area of the ſegment BDC. *Q. E. F.*

Prob. LXVII.

To conſtruct an *oval* or *ellipſis*.

As I have not hitherto ſaid any thing of this figure, it will be neceſſary here to explain it, before it is conſtructed.

An

An oval *or* ellipſis, *is a geometrical figure of a curved form, having one of its diameters, as* C D (fig. 68) *longer than the other* E F. *The longeſt diameter* C D *is called the* tranſverſe, *and the ſhorteſt* E F, *the* conjugate. *The two points* H *and* G, *in the tranſverſe diameter, are called the* focus, focii, *or* centers; *and the point* O, *where the two diameters interſect each other, is called the* middle point, *or* common center *of the ellipſis. Any lines croſſing the tranſverſe diameter at right-angles, as* a d, f e, *&c. are called* ordinates *to the ellipſis. The half* i a, *or* H f, *of the ordinate, is called the* ſemi-ordinate, *and is frequently conſidered as the ordinate itſelf. The ordinates* e f *and* g h, *which paſs through the focal points of the tranſverſe diameter, are called the* latus rectum, *that is, the* ordinate rightly applied. *The diſtance* C i, C H, *&c. on the tranſverſe diameter, as alſo* D k, DG, *&c. are called* abſciſſæ, *that is,* parts cut off. *But when the rectangle of any two abſciſſæ is taken, they muſt be ſuch as, when added together, their ſum will equal the whole tranſverſe diameter.* Theſe things being known, conſtruct the *oval* as follows:

231. *Let the* oval *be that at* fig. 67, No. 1.

Take, at pleaſure, any two points, as F, G, for the centers, in which ſtick two pins, and let a ſmooth ſtring be provided in length, when doubled, more than the diſtance F G; tie the two ends of this line together, and, laying it over the pins, carry the loop of it round them, drawing it tight by help of a pencil in the loop: the pencil during this motion, if kept even in the loop, will deſcribe the oval A B C D E I. *Q. E. F.*

Otherwiſe thus. fig. 67, No. 1.

Deſcribe the two circles A B G and F C D, with the ſame radius paſſing through the centers of each other in F and G, theſe will cut each other in the points *a* and *b*; from *b* and *a*, as centers, and with the radius A G or F D, ſweep the arches B C and I E: theſe arches, with the remaining arches of the circles I A B and C D E, will compleat the oval. *Q. E. F.*

232. Or

232. Or thus, Suppoſe the diameters were given, that is, the *oval* to be of a given length and breadth.

Let the tranſverſe diameter be A, *and the conjugate diameter* B. fig. 67, No. 2.

Take the line C D equal the tranſverſe diameter A, on which lay from C to G the conjugate diameter B; divide the remainder of the tranſverſe diameter G D into three equal parts (145) and biſect C D in O, on each ſide of which lay off two-thirds of G D, which will give the focal points G and H. On the line G H make the equilateral triangles H E G and H F G, and produce their ſides both ways, from G and H towards the points *a*, *b*, *c* and *d*; on H and G, as centers, and with the radius H C or G D, ſweep the arches *d* C *a* and *b* D *c*, and from the points E and F, with the radius F *a* and E *d*, ſweep the arches *a* E *b* and *b* F *c*, ſpringing from the former arches in *a*, *b*, *c* and *d*, which will compleat the oval. *Q. E. F.*

Prob. LXVIII.

233. To meaſure the area of an *ellipſis*, or *oval*.

Let the given ellipſis *be* C E D F. fig. 68.

Geometrically. Find a mean proportional A B between the diameters (196) which will be the diameter of a circle; the area of which being found *(per* 226) will be the area of the ellipſis required. Or thus,

Arithmetically. Multiply the product of its two diameters by the decimal number .7854 *, and the

* This number is the area of a circle, the radius of which is unity, or 1.

product

product will be the area required. But if the area of a ſegment be required, and the baſe parallel to the tranſverſe diameter, find the areas of the inſcribed circle and of the whole ellipſis, and uſe the following proportion:

: the *area* of the circle :: the *area* of the ellipſis, :: the *area* of the *ſegment* in the inſcribed circle (cut by the baſe of that in the ellipſis) : the *area* of the *ſegment* in the ellipſis.

Prob. LXIX.

234. To conſtruct an *oval* or *ellipſis* mechanically. fig. 67.

Method 1. Make a ſquare, as ABCD, of any convenient ſize, to anſwer the purpoſe deſigned.

2. Produce the ſides (as *per* figure) to a convenient length, as to *a*, *b*, *c* and *d*.

3. From the point A, as center, and with the interval *d* A, deſcribe the arch *d c*; in the ſame manner from C, as center, with the radius C *b*, deſcribe the arch *a b*.

4. From B, as center, with the interval B *c*, deſcribe the arch *c a*; and from D, in the ſame manner, with the ſame radius, or the interval D *d*, deſcribe the arch *a b*, which will compleat the oval. *Q. E. F.*

Prob. LXX.

235. To conſtruct a *parabola*.

Let the axis CD, *and the ordinate* AB, *be given.* fig. 68.

Before this can be done, it will be neceſſary to ſay ſomething concerning the parabola. *It is therefore a geometrical figure, produced from the ſection or the cutting of a cone parallel to*

one

one of its ſides, that is, the plane ſurface produced from this ſection, is called a parabola. *And here obſerve, that the* cone *which produces this figure is a ſolid body, the baſe of which is a circle, the remainder of its ſurface being alſo circular, and terminating in a point perpendicularly over the center of its baſe; ſo that in form it is nearly like a ſugar-loaf; and the* axis *of the* parabola *a right-line perpendicular to, and biſecting any ordinate; and the* ordinate, *like that in the ellipſis, is any right-line, croſſing the axis at right-angles, and terminating in the curve of the* parabola. *This being known, you may conſtruct it as follows:*

Take the line A B equal the given ordinate, which biſect in C; on C raiſe a perpendicular C D, equal to the given axis, which produce below A B, until it meets a circle, deſcribed through the points A, D and B (164) in *a*. Upon the line C D deſcribe as many circles as you pleaſe, having their centers in that line, each paſſing through the point D; theſe circles will cut C D in the points *b*, *c*, *d*, &c. Let C *a* be taken, and transferred upwards towards D, from the points of interſection *b*, *c*, &c. to the points *m*, *n*, *o*, &c. through the points *m*, *n*, *o*, &c. draw chords parallel to A B, and where theſe chords cut their circles, the curve of the *parabola* muſt paſs; therefore, let them be joined by a careful ſteady hand, and the *parabola* will be compleated. *Q. E. F.*

PART

PART III.

AXIOMS.

1. WHAT things or quantities, are equal to one and the ſame thing or quantity, are equal among themſelves.

2. If equal things be added to equal things, the ſums or wholes will be equal.

3. If from equal things equal things be taken away, the remainders or differences will be equal.

4. If to unequals you add equals, the ſums or wholes will be unequal.

5. If from unequals you take away equals, the differences or remainders will be unequal.

6. What things are double or treble, the ſame things are equal.

7. Thoſe things which, being applied to, or laid upon, each other, and found to agree in all particulars, are equal. *(See rem. def.* 76).

8. If right-lines or angles be equal, they will mutually agree with each other; and the contrary. *(See rem. to def.* 5 *and* 25).

9. Every whole thing or quantity is greater than its part, *for, the whole is equal to all its parts together.*

10. All right-angles are equal among themſelves.

11. Equal circles have equal radii.

12. Two right-lines cannot include a ſpace, becauſe, for that purpoſe, three right-lines at leaſt will be requiſite.

When the reader has committed the above axioms well to memory, let him make himſelf acquainted with the following abreviations or characters, made uſe of in the following work, together with thoſe formerly explained at the latter end of the firſt part.

∠ *Angle*, □ *Square*,
∟ *Right-angle*, ∥ *Parallel*,
△ *Triangle*, ▭ *Parallelogram*,
⊥ *Perpendicular*, ∴ *Therefore*,
i. e. That is.

AB^2 *The ſquare of any quantity repreſented by* AB,

1 2 3 *Figures thus placed in the margin, ſignify the number of the ſtep or argument made uſe of in the ſynthetical demonſtration of any theorem, &c. Def.* 135.

Hpo. *Hypotheſis*, Cor. *Corollary*,
Rgle. *Rectangle*, Def. *Definition*,
Ax. *Axiom*, Conſeq. *Conſequently*.

Q. E. D. Which was to be demonſtrated.

(1 and 2, &c.) *Figures thus joined by the word* and, *or ſingly, are uſed as a reference to the particulars proved in the ſtep or ſteps refered to by the numbers.*

THEOREMS.

Theo. 1. 4 Euc. Lib. 1.

If two ſides AB, AC, *and the included* ∠ A *in one* △ ABC, *be* = *to the two ſides* DE, DF, *and the included* ∠ D *of any other* △, DEF, *each to each; the remaining ſide or baſe* BC *of the one, will be* = *to the remaining ſide or baſe* EF *of the other. Again, the* ∠*s* B *and* C *at the baſe of the one, will be* = *to the* ∠*s* E *and* F *at the baſe of the other: and laſtly, the whole* △*s will be congruous,* (76) *conſequently* = *to each other.*

Hypothesis.	Thesis.
In the △*s* ABC & DEF,	1 BC = EF.
AB = DE, AC = DF,	2 ∠B = ∠E, & ∠C = ∠F.
and ∠A = ∠D.	3 △ABC = △DEF.

Demon-

DEMONSTRATION.

1 LET the point A be laid upon the point D, (141) then from the equality of the lines A B and D E (*by Hpo.*) the point B will fall upon E, and the lines agree with each other. (*Ax.* 8). And becauſe ∠ A = ∠ D (*by Hpo.*) the line A C will fall upon D F; and, as the lines are =, the point C will fall upon F. Hence the three points A, B and C of the one Δ, will exactly fall upon, and agree with the three points D, E and F of the other Δ; conſeq. the ſide B C will perfectly agree with the ſide E F, ∴ they will be = to each other (*by Ax.* 7). *Q. E. D. in the firſt part.* Again,

2 Becauſe the three points and ſides of the Δ A B C, fall exactly upon, and agree with theſe of the Δ D E F (*by* 1 *ſtep*) the ∠ B will fall upon, and agree with the ∠ E; and alſo, in the ſame manner ∠ C will agree with ∠ F; ∴ the ∠s will be = (*by ax.* 7). *Q. E. D. in the ſecond part.* And,

3 Perceiving from the arguments uſed in the foregoing ſteps, that all the ſides and ∠s of the Δ A B C, agree perfectly in every reſpect with the ſides and ∠s of the Δ D E F; the Δs ∴ muſt be = and congruous. (*per ax.* 7, *and def.* 76) *Q. E. D. in the third part.*

THEO. 2. SCHO. 4. EUC. LIB. 1.

If in two Δ*s* A B C *and* D E F *there be two* ∠*s* A *and* B *and the included ſide* A C *in one* Δ, = *to two* ∠*s* D *and* E, *and the included ſide* D E, *in the other, all the other parts will be* = *and the* Δ *congruous.* (76) fig. 2.

HYPOTHESIS.

In the Δ*s* A B C *and* D E F, ∠ A = ∠ D *and* ∠ B = ∠ E, *and* A B = D E.

THESIS.

Δ A B C = Δ D E F.

DEMON-

DEMONSTRATION.

1 LET the point D be laid upon the point A, (141) and the ſide D E upon A B, which, becauſe they are = (*by Hpo.*) the point E will fall upon B. And, from the equality of the ∠ E to the ∠ B, the ſide E F will fall upon B C. Now,

2 Suppoſing the ſide E F greater or leſs than the ſide B C, in either caſe the point F, of the △ D E F, will not fall upon the point C of the other △, from which ſuppoſition the other ſide D F, will not fall upon A C, conſeq make the ∠ D greater or leſs than the ∠ A. But,

3 Becauſe the ∠ D is (*by Hpo.*) = to the ∠ A, the line D F ∴ muſt fall upon the line A C; and conſequently the point F on the point C; *i. e.* the ſides of the △ D E F will fall upon and agree with the ſides of the △ A B C, from which agreement they will be = and congruous. *Q. E. D.*

THEO. 3. 8 EUC. LIB. 1.

If in two △*s* ABC *and* DEF, *all the ſides in one* △ *be* = *to all the ſides in the other, each to each reſpectively, the* ∠*s oppoſite to the* = *ſides will be* =, *and the whole* △*s* = *and congruous.* (76) fig. 3.

HYPOTHESIS.	THESIS.
In the △*s* ABC, & DEF, AC=DF, AB=DE, & BC=EF.	1 ∠A=∠D, ∠B=∠E, & ∠C=∠F. 2 △ABC=△DEF.

DEMONSTRATION.

1 LET the point D be laid upon, or applied to the point A, and the line DE to A B, (141) which, becauſe they are = (*by Hpo.*) the point E will fall upon the point B. Then

2 On A, with the radius AC deſcribe the arch *a b* (139), and, becauſe the ſide D F = A C (*by Hyp.*) the point F will

F will fall ſomewhere in that arch; in the ſame manner, with the ſide BC, from B as center, deſcribe the arch *d e*, cutting *a b* in C, and in which arch the point F muſt alſo fall, becauſe EF = BC. But,

3 From the equality of the lines AC to DF, and BC to EF (*by Hyp.*) the point F muſt ∴ fall on, or be on both circles, *i. e.* in the point C where they interſect ∴ as they are found to agree in every particular ∠ to ∠, and ſide to ſide, the ∠ A will = ∠ D, ∠ B = ∠ E, &c. *Q. E. D. in the firſt part.* And laſtly,

4 From this agreement of the △s, they muſt be = and congruous. *Q. E. D. in the ſecond part.*

THEO. 4. 26 EUC. LIB. 1.

If in two △*s* ABC *and* DEF *the* ∠*s* A *and* B, *and the ſide* AC *oppoſite one of them in one* △, *be reſpectively* = *to the two* ∠*s* D *and* E, *and the ſide* DF *oppoſite to an* = ∠ *in the other, the whole* △*s will be* = *and congruous.* (76) fig. 4.

HYPOTHESIS. THESIS.

The ∠A = ∠D, ∠B = ∠E, *and* AC = DF. △ABC = △DEF.

DEMONSTRATION.

1 LET the point A of one △ be laid upon the point D of the other, (141) then from the equality of the line AC to DF (*by Hpo.*) the point C will fall upon F. And,

2 Becauſe ∠ A = ∠ D, the line AB will fall upon DE; but, as AB is not given = to DE, the point B may not, in that caſe, fall upon E, but, on ſome point between D and E, or beyond E, as in *g*, and ∴ make the ∠ D *g* F = ∠ ABC. But,

3 The ∠ DEF = ∠ ABC (*by Hpo.*) conſeq. the ∠ D *g* F will = ∠ DEF (*per Ax.* 1) *i. e.* a part = to the whole; which being abſurd, and contradictory to the 9th axiom, ∴ the point B, as it cannot fall either

either on one ſide or the other of the point E, muſt evidently fall upon E. Now,

4 Becauſe ∠ E = ∠ B *(by Hpo.)* the ſide B C muſt fall upon E F, and make the two △s agree in every particular; conſequently be = and congruous. *Q. E. D.*

THEO. 5. 5 EUC. LIB. 1.

In an iſoſceles or equicrural △, *as* A B C, *the* ∠*s at the baſe* A *and* B, *oppoſite to the* = *ſides* A C *and* B C, *will be* =.

HYPOTHESIS. THESIS

The △ A B C *is an iſoſceles* △. ∠ A = ∠ C.

To prove this, you muſt beſect the line A B in *d*, (144) and draw the line *d* C, which will divide the △ into the two △s A *d* C and B *d* C.

DEMONSTRATION.

1 I SAY, theſe △s A *d* C and B *d* C are congruous, for, A *d* = *d* B *(by conſtruction)* A C = BC *(by Hpo.)* and C *d* is common to both. ∴

2 ∠ A will = ∠ B, being oppoſite to the common and conſeq. = ſide *d* C *(per Theo.* 3, *or* 8. *E. lib.* 1.) *Q. E. D.*

COROLLARY 1.

HENCE, a right-line let fall from the vertex of an iſoſceles △, ſo as to beſect the baſe, it will alſo beſect the vertical ∠, and be ⊥ to the baſe; *for the* △ A *d* C *is equilateral to the* △ B *d* C, *conſeq. the* ∠*s at* C *being oppoſite to the* = *lines* A *d and d* B, *are* =; *and, as the* ∠*s at d are* A C and B C; *oppoſite to the* = *ſides;* ∴ *d* C *muſt be* ⊥. *(Per. Teo.* 3, *or* 8. *E. Lib.* 1.)

COR. 2. In any right-lined △, when the ∠s at the baſe are =, the ſides oppoſite thoſe = ∠s will alſo be =, *for, the two* △*s* A *d* C *and* B *d* C *are congruous,*

gruous, conſeq. by any of the foregoing theorems, they will agree ∠ to ∠, and ſide to ſide. ∴ &c. 6 Euc. Lib. 1.

Theo. 6. 13 Euc. Lib. 1

If any right-line B C *ſhould fall upon another right-line* A B, *it will either make two* ⌞*s, or* ∠*s* = *to two right ones.* fig. 6.

Hypothesis.	Thesis.
That BC *is a right-line, meeting or ſtanding on the right-line* A D.	1 ∠ A C D & ∠ B C D *are both* ⌞*s. or* 2 *they are* = *to* 2 ⌞*s.*

To prove this, from the point C, as center, ſweep the ſemicircle A B D (139) and raiſe the ⊥ C *d.* (146).

Demonstration.

1 LET the line B C ſtand ⊥ on A B (*i. e.* in the poſition C *d*); in that caſe, the ∠s on each ſide would be ⌞s *(per def.* 33) *Q. E. D. in the firſt part.* Again,

2 Let the line BC ſtand obliquely, (as it appears to do in the figure) in that caſe the arch B D will be the meaſure of the ∠ *c (def.* 29) and in like manner will the arch B *d* A be the meaſure of the ∠ A C B, or the ſum of the ∠ *a* and ∠ *b* together. But,

3 The arch B D + the arch B *d* A = a ſemicircle, and as the ſemicircle will meaſure two ⌞s *(per rem. def.* 30) the ∠ A C B + ∠ B C D will = 2 ⌞s. *Q. E. D. in the ſecond part.*

Corollary 1.

HENCE, if ever ſo many right-lines ſhould fall upon another right-line in the ſame point, the ſum of all the ∠s taken together will only = two ⌞s; *for, in that caſe, it will be two* ⌞*s divided into ſeveral parts, or, a ſemicircle will meaſure them all.* (See rem. def. 30.)

Cor. 2. If ever ſo many right-lines croſs or interſect each other in the ſame point, all the ∠s about that

that point, taken together, will only = four ∟s; *for, if a circle be ſwept about the point of interſection, as center, the circumference will meaſure them all.* ∴ *&c.* (See rem. def. 30).

THEO. 7. 15 EUC. LIB. 1.

If two lines AC *and* BE *interſect each other, the vertical or oppoſite ∠s will be* =. (70)

fig. 7.

HYPOTHESIS.	THESIS.
The right-lines AC *and* BE, *interſect, or cut each other.*	$\angle a = \angle b$ *or* $\angle c = \angle d$.

DEMONSTRATION.

1 I SAY, the $\angle a + \angle c = 2$ ∟s. (*per theo.* 6. *or* 13 *E. Lib.* 1.) And,

2 The $\angle c + \angle b = 2$ ∟s. (*per. ſame.*) Conſeq.

3 The $\angle a + \angle c = \angle c \times \angle b$. (*per ax.* 1) But

4 The $\angle c$ is a common $\angle$ on both ſides of the equation, (in 3d. ſtep,) ∴ taking it away, there will remain the $\angle a = \angle b$ (*per ax.* 3). *Q. E. D.*

In the ſame manner may the $\angle c$ be proved = $\angle d$.

THEO. 8. 27 EUC. LIB. 1.

If a right-line EF, *cut two* || *lines* AB *and* CD, (39) *the external ∠* a *will be* = *to the internal and oppoſite one* d, *on the ſame ſide of the cutting line* EF; (61) *again, the alternate ∠s* b *and* d *will be equal;* (69) *and laſtly, the two internal ∠s* f *and* b, *or* e *and* d, (62) *on the ſame ſide of the cutting line, will, together* = *two right-angles.*

HYPO-

HYPOTHESIS. THESIS.

That A B *and* C D *are* || *lines, and cut by the right-line* E F.

1 *External* ∠ *a* = *the internal* ∠ *d*
2 *Alternate* ∠ *b* = *the alternate* ∠ *d*
3 *The internal* ∠ *s b and f, or d and e are on the same side of* E F = 2 ∟ *s.*

DEMONSTRATION.

1 I SAY, because the lines A B and C D are || (*per Hyp.*) the line E F will have the same inclination to both (*per def.* 39). But

2 This inclination is expressed by the ∠s *a* and *d*, ∴ the external ∠ *a* = internal ∠ *d*. *Q. E. D. in the first part.* Again,

3 The ∠ *a* = the ∠ *b*. (*per theo.* 7. & 15 *E. lib.* 1.) Also,

4 The ∠ *a* = the ∠ *d*, (*per 2d step*) ∴ the alternate ∠ *b* = the alternate ∠ *d*. (*per ax.* 1.) *Q. E. D. in the 2d part.*

In the same manner may the ∠ *e* be proved, = ∠ *f*. Lastly,

5 The ∠ *a* + ∠ *e* = 2 ∟ s (*per. theo.* 6. & 13 *E. lib.* 1.) But,

6 The ∠ *a* = ∠ *d*. (*per 2 step*). ∴

7 The ∠ *d* + ∠ *e* must equal 2 ∟ s, (*per. Ax.* 1.) *Q. E. D. in the third part.*

In the same manner may the ∠ *b* + ∠ *f* be proved, = 2 ∟ s.

COROLLARY 1.

HENCE, if a right line cuts two other right lines, and make the external ∠ equal to the internal and opposite ∠, or the two internal ∠s on the same side = to two right ones, the lines will be parallel. (29 *E. Lib.* 1.)

COR. 2. If one right line cut two other right lines, and make the alternate ∠s equal, then those lines will be parallel. (28 *E. Lib.* 1.)

THEO. 9. 32 EUC. LIB. 1.

In any △ *as* A B C, *the external* ∠ C B E *is equal to the two internal remote ones* A *and* C, *and the sum of the three angles together will* = 2 ∟ *ones.*

HYPOTHESIS. THESIS.

That the fig. ABC *is a* Δ *and its ſide* AB *is produced to* E.

1 *The external* ∠ CBE = *the two internal and oppoſite* ∠*s* a *and* c.
2 ∠ *a* + ∠ *c* + ∠ *b* = 2 ∟*s*.

To prove this, through the point B, draw the line DB ‖ to AC. (153).

DEMONSTRATION.

1 I SAY, the ∠ *d* = ∠ *a*. *(per part* 1. *theo.* 8. *and* 37 *E. lib.* 1.) And,

2 The ∠ *e* = ∠ *c*. *(per part* 2, *of the ſame).* But,

3 The ∠ *e* + ∠ *d* = the external ∠ CBE. *(per. ax.* 9). ∴ the external ∠ CBE is = the internal ∠s *a* and *c* together. *Q. E. D. in the firſt part.* Again,

4 The external ∠ CBE + ∠ *b* = 2 ∟s. *(per theo.* 6. *and* 37 *E. lib.* 1). But,

5 The ∠ CBE = ∠ *a* + ∠ *c*. *(per the firſt part of this.)* ∴ the ∠ *a* + ∠ *c* + ∠ *b*, will equal 2 ∟s. *(per* 4 *and* 5.) *Q. E. D. in the ſecond part.*

COROLLARY 1.

THE external ∠ of any Δ, is greater than either of the two internal oppoſite ones; *for, it is = to them both together.*

COR. 2. In any Δ, if one of the ∠s be right or obtuſe, the other two will be acute; *for, they are all together* = 2 ∟*s*.

COR. 3. If two ∠s of one Δ be = to two ∠s of any other Δ, the remaining ∠ in both Δs will be =, *for, the ſum of the two* ∠*s in one,* + *the third* ∠ *is* = *to the ſum of two* ∠*s in the other,* + *the third* ∠.

COR. 4. If one ∠ in one Δ, be = to one ∠ in any other Δ, the remaining two ∠s in both Δs will be =, *for the ſum of all the* ∠*s in one, are* = *to the ſum of all the* ∠*s in the other.*

COR. 5. If one ∠ in any Δ be a right one, the remaining two together will make one right one.

COR. 6. When in an iſoſceles Δ, the ∠ included between the = ſides is a right one, the other two ∠s

are

are each of them half right ones; *for, they are = to each other. (per theo. 5, and 5 E. lib. 1), and, are together equal to one right one. (per cor. 5, of this.)*

Cor. 7. In an equilateral Δ, each angle is $\frac{2}{3}$ of one right one; *for, as it is* $\frac{1}{3}$ *of two right ones ∴ it muſt be* $\frac{2}{3}$ *of one.*

Theo. 10. Part of 32 Euc. Lib. 1.

In every quadrilateral figure, as ABCD, *the four ∠s taken together will = four ∟s.*

Hypothesis.	Thesis.
The fig. ABCD *is a quadrilateral.*	∠A + ∠B + ∠C + ∠D = *four* ∟*s.*

To prove this, draw the diagonal AC, which will divide the figure into two Δs.

Demonstration.

1 THE ∠s in each Δ taken together, are = 2 ∟s, (*per theo.* 9. *and* 32 *E. lib.* 1.) And,

2 Since all the ∠s in the two Δs, together = the ∠s in the whole quadrilateral ABCD; then, if the ∠s of one Δ, be added to thoſe of the other, the ſum muſt = the ∠s in the quadrilateral. But,

3 The ∠s in the two Δs taken together, are = to four ∟s; ∴ all the ∠s in the quadrilateral ABCD will = four ∟s. *Q. E. D.*

Theo. 11. Part of 32 Euc. Lib. 1.

In any polygon whatever, all the internal ∠s taken together, will = twice as many ∟s as the polygon hath ſides, abating four.

Hypothesis.	Thesis.
That the figure A *is a polygon.*	*All the internal ∠s will = twice as many ∟s, as there are ſides in the polygon, abating four.*

To

To prove this, take any point within the polygon as A, from which, draw right lines to every ∠ in it, and theſe lines will divide it into as many △s, as the polygon hath ſides.

DEMONSTRATION.

1 EACH △ contains two ∟s. (*per theo.* 9. *and* 32 *E. lib.* 1.) conſeq. all the ∠s in each △ taken together, will = twice as many ∟s, as the polygon hath ſides. But,

2 In theſe △s, there are four ∟s about the point A, (*per theo.* 7. *and* 15 *E. lib.* 1.) which have nothing to ſay to the ∠s of the polygon. ∴ if you take theſe four ∟s away, the remaining ∠s which alone belong to the polygon, will together, = twice as many ∟s as the polygon hath ſides, abating four. Q. *E. D.*

THEO. 12. Part of 32 EUC. LIB. 1.

All the external ∠s of any polygon whatever, will = only four ∟s.

HYPOTHESIS. THESIS.

The figure A *is a polygon, the ſides of which are produced.* *All the external ∠s together = only four ∟s.*

DEMONSTRATION.

1 EACH external ∠, with its adjacent internal one, will = 2 ∟s. (*per theo.* 6. *and* 13 *E. lib* 1.) Conſeq.

2 All the internal and external ∠s together, will = twice as many ∟s, as the polygon hath ſides. But

3 The internal ∠s together, = twice as many ∟s as the polygon hath ſides, abating four. (*per theo.* 11. *and* 32 *E. lib.* 1.) ∴

4 Taking away the internal ∠s, the remaining external ∠s, will = only four ∟s. Q. *E. D.*

COR.

COROLLARY.

HENCE, the external ∠ of any regular or ordinate polygon, is always = to the ∠ at the center of it; *for, the external ∠s together, are in number = to the ∠s at the center, which, being = to four* ∟*s, consequently, whatever part of four* ∟*s the* ∠ *at the center is, the same part must the external* ∠ *be,* ∴ &c. But *e. g.* if the polygon be a *pentagon*, the external ∠ would be the $\frac{1}{5}$ of 360°, or four ∟s, *i. e.* 72°. Again, if an *hexagon* 60°, if an *octagon* 45°, a *nonagon* 40°, a *decagon* 36°, and, if an *eptagon* 51 $\frac{3}{7}$ or 51 $\frac{1}{2}$, nearly because 360° cannot be exactly divided by 7.

Note, the conclusion to be drawn from this theorem is amazing! for, suppose a polygon to be constructed with 1000 *sides, (or as many more as you please) it would have as many external ∠s; and notwithstanding their number would be so great, their contents, added together, will make only four* ∟*s.*

THEO. 13. 18, and 19 EUC. LIB. 1.

In every right lined △ *as* ABC, *the greatest* ∠ A, *is opposite to the greatest side* BC; *and the contrary, i. e.* the greatest side CB is opposite to the greatest ∠ A. fig. 13.

HYPOTHESIS.

In the △ ABC *the side* CB *is greater than either* AB, *or* AC; *and the* ∠ A *is greater than either the* ∠ B, *or the* ∠ C.

THESIS.

1 *The* ∠ A *opposite to the* greatest *side* CB, *is* greater *than the* ∠ C, *opposite to the lesser side* AC.

2 *The* greatest side CB *is opposite to the* greatest ∠ A.

To prove this, make BD = BA (140) and draw AD, which will make the △ ABD an isoceles △. (45).

DEMONSTRATION.

1 BECAUSE, BD = BA by construction, the ∠ *a* will = ∠ *b*. (*per theo.* 5. *and* 5 *E. lib.* 1). Again,

2 The

2 The $\angle b$ is greater than the $\angle$ C. (*per cor.* 1 *to theo.* 9). But

3 The $\angle$ CAB is greater than the $\angle a$, (*per ax.* 9.) which is equal the $\angle b$. (*per conſtruction.*) ∴

4 The $\angle$ CAB, oppoſite to the greateſt ſide CB, is greater than the $\angle$ C, &c. *Q. E. D. in the firſt part.*

In the ſame manner, may the $\angle$ A be proved greater than the $\angle$ B.

5 The converſe of this theorem muſt be true from the firſt part; for, if it were not the caſe, CB, the greateſt ſide, muſt be oppoſite to the leſſer $\angle$ C or B. But,

6 The greateſt ſide CB is oppoſite to the $\angle$ A, which, in the firſt part was proved to be the greateſt $\angle$. ∴ &c. *Q. E. D. in the ſecond part.*

THEO. 14. 34 EUC. LIB. 1.

In every □ ABDC *the oppoſite* $\angle$*s, and oppoſite ſides are* =, *and, it is cut into two* = *parts or* $\triangle$*s by the diagonal* BC.

HYPOTHESIS.

The figure ABDC *is a* □ *and* BC *is a diagonal line.*

THESIS.

1 $\angle$ B = $\angle$ C & $\angle$ A = $\angle$ D

2 *The ſide* AB = CD & AC = BD.

3 *The* $\triangle$CAB = *the* $\triangle$CDB.

DEMONSTRATION.

1 BECAUSE AB and CD are ‖ (*per def.* 31.) the $\angle a = \angle b$ (*per. theo.* 8. *and* 27 *E. lib.* 1), being alternate, in the ſame manner is the $\angle c = \angle d$. (AC and BD being ‖.) Conſeq. the $\angle a + \angle c$, will = $\angle b + \angle d$. (*per ax.* 2). But,

2 The $\angle$ B = the $\angle a + \angle c$, and the $\angle$ C = the $\angle b + \angle d$. (*per ax.* 9). ∴ the $\angle$ B = $\angle$ C. *Q. E. D. in the firſt part.* Again,

3 In the $\triangle$s CAB and CDB, the $\angle$s *a* and *d* in one $\triangle$ are = to the $\angle$s *b* and *c* in the other (*per* 1 *ſtep*) and the ſide BC between thoſe $\angle$s, is common to both $\triangle$s, conſeq

conſeq. they are congruous. (*per theo.* 2. *and Scho.* 4. *E. lib.* 1.) ∴ the ſide AB = CD, and the ſide AC = BD, (*per theo.* 2. *and Scho.* 4. *E. lib.* 1). *Q. E. D. in the ſecond part.* Laſtly,

4 The △ CAB = △ CDB; becauſe, in the ſecond part they were proved to be congruous, ∴ they muſt be = and conſeq. halves of the □. *Q. E. D. in the third part.*

COROLLARY 1.

HENCE, if any point as *o*, be taken in the diagonal of any □, and, there be drawn through it, the lines *q r* and *s t* || to the ſides of the □, the complements *q s* and *r t* (56) will be =; *for, the whole* □ AD *is divided into two* = *parts; by the diagonal* BC, *viz. the* △*s* CAB *and* CDB, *and into four* □*s by the lines* q r *and* s t, *two;* q t *and* s r *about the diagonal, and two,* q s *and* t r *the complements of thoſe to the whole* □; *but, the* □*s* q t *and* s r *about the diagonal, are equally divided by the line* BC *(it being a diagonal to both)* ∴ *taking away their halves on each ſide of this diagonal, from the whole* △*s* CAB *and* CDB, (*which, by the third part of this theorem are* =) *the remainder,* viz. *the* □*s* q s *and* t r *muſt be* = *alſo.* (*per ax.* 3). 43 E. lib. 1.

THEO. 15. 35 and 36 EUC. LIB. 1.

The □*s* EABF, ECDF *and* GCDH *ſtanding upon the ſame or* = *baſes* EF *and* GH, *and between the ſame or equidiſtant* ||*s*, AD *and* FH, *are* = *to each other.*

HYPOTHESIS.

That the quadrilaterals EABF, ECDF *&* GCDH *are* □*s upon the ſame or* = *baſes,* EF *&* GH; *&* *between the ſame* ||*s* AD *and* FH.

THESIS

The □*s* EABF, ECDF *&* GCDH *are* =.

To prove this, draw the lines FA, and FC. (137).

DEMON-

DEMONSTRATION.

1 I SAY, the △ AEC = △ BFD (*per theo.* 1. *and* 4 *E. lib.* 1.) *for*, AE = BF (*per theo.* 14. *and* 34 *E. lib.* 1.) and AC = BD (*per rem. to def.* 67.) becauſe, AB = CD, and CB is a common line added; alſo, the ∠s ACE and BDF between thoſe = ſides are =. (*per part* 1. *theo.* 8. *and* 27 *E. lib.* 1). But,

2 The △ B*e*C is common to both theſe = △s AEC and BFD, conſeq. taking it away from both, the remaining parts or trapeziums, AB*e*E and CDF*e* will be =. (*per ax.* 3.) Now,

3 If to thoſe = trapeziums, you add the △ E*e*F, the wholes, *i. e.* the □ EABE and ECDF will be =. (*per ax.* 2.) *Q. E. D.*

In the ſame manner may the □ GCDH be proved = ECDF, ∵ the three □s are =. (*per ax.* 1.)

COROLLARY 1.

HENCE, the △s EAF, ECF and GCH ſtanding upon the ſame or = baſes EF and GH, and between the ſame ‖ s AD and EH are = : *for*, (per theo. 14, and 34 E. lib. 1.) *they are halves of the* □*s, on whoſe baſes they ſtand; which, by the hypo. in this theo. are* =, ∵ *if the wholes* (*i. e. the* □*s*) *be* =, *the halves of the wholes*, (*i. e. the* △*s*) *will be* = *alſo.* 37 E. lib. 1.

COR. 2. Hence alſo, if a △ ſtand between the ſame ‖ s with any □, and have the ſame or an = baſe with it, it will be half of the □ : *for, the* △ ECF *is* = *to the* △ EAF, (per cor. 1. & 37 E. lib. 1.) *but, the* △ EAF *is half the* □ EB (per theo. 14. & 34 E. lib. 1.) ∵ *the* △ EAF *is alſo half of the* □ EB. 41 E. lib. 1.

In the ſame manner may the △ GCH be proved = to half the □ EB.

COR. 3. Hence alſo, any △ ſtanding between the ſame ‖ s with a □, having its baſe double that of the □, the △ muſt be = to the □: *for, if the baſe of the* △ *was* = *to that of the* □, *the* △ *would be* = *one half the* □ ; *but, being double, the* △ *muſt be* = *the* □. 42 E. lib. 1.

Note,

Note, from the above theorem let it be obſerved, that the perimeter (87) of any figure is inſufficient to determine its area; for, were the parallel lines A B and E H, produced to any conſiderable length or diſtance, the □ B E would be = in area, to any □ ſtanding upon the ſame or an = baſe with it, and between the ſame ‖ s, be the other □ of what extent or obliquity you pleaſe; and here it may be farther obſerved,

That the greateſt part of mankind form their idea of the ſize of any ſpace, as a town or city, from the circumference of it, *i. e.* its perimeter. Now, were ſuch men to be informed that any town, the circumference of which meaſured only 500 chains, perches, &c. contained an = area with another town, which meaſured in its circumference 5000 chains, perches, &c. or, that the area of the former, was three times as great as that of the latter, they would be ſtruck with amazement; but, when once made acquainted with a knowledge of the above theorem, their wonder muſt ceaſe, and, while they give up their opinion, admire the principles of that ſcience, which reconciles (to them) ſuch apparent impoſſibilities.

Theo. 16. 47 Euc. Lib. 1.

In every ∟*'d* △ *as* ABC, *the ſquare of the hypothenuſe* BC, *or the ſide oppoſite to the* ∟, *is* = *to the ſum of the ſquares on the other two ſides* BA, *and* AC.

fig. 16. No. 1, and 2.

Hypothesis. *That* A B C *is a* ∟*'d* △, ∟*'d at* A.

Thesis. $BC^2 = AB^2 + AC^2$

To prove this, let ſquares be conſtructed upon each ſide of the △ (156) and poſited as in the fig. No. 1; draw alſo the line F G ‖ to D C or E B, and from the point A draw A E, A D; and laſtly, produce I H unto D.

DEMONSTRATION.

1 The □ made upon BC is = the two □s BF and DG, (*both from conſtruction and ax.* 9.) Again,

2 The △ BEA is = to half the □ upon BA, *i. e.* to the □ BK, and alſo = to half the □ BF, (*per cor.* 1, *theo.* 15, *and* 37 *E. lib.* 1.) Conſeq.

3 The □ BK = □ BF, (*per ax.* 6) in the ſame manner is the □ upon AC, *i. e.* CH = □ GD, *for, the* △ ADC *is half of each,* (*per cor.* 1, *theo.* 15, *and* 37 *E. lib.* 1.) But,

4 The □ BK is the □ made upon AB, and the □ CH is that which is made upon AC. ∴

5 The □ BD made upon BC, (or BC^2) is = to the □s BK and CH together, *i. e.* to the □s which were made upon BA and AC; or $BC^2 = AB^2 + AC^2$. *Q. E. D.*

Another Proof more ſimilar to Euclid.

Let the □s be conſtructed as in fig. 16. (No. 2.) upon the ſides of the △ ABC, and draw BE ‖ to AF or CR, and alſo the other lines IC and BF.

1 The ∠ IAB = the ∠ FAC, being both ∟s (*per ax.* 10.) now, if to theſe right and = ∠s you add the common or intermediate ∠ BAC, then will the ∠ FAB = ∠ IAC. (*per def.* 64, *and ax.* 2.) Again,

2 The △ IAC = △ FAB (*per theo.* 1, *and* 4 *E. lib.* 1.) for, the lines IA, AC, and the included ∠ IAC in one △, is = to the lines BA, AF, and the included ∠ FAB, in the other. Alſo,

3 The △ IAC = half □ AL, which was made upon AB (*per cor.* 1, *theo.* 15, *and* 37 *E. lib.* 1.) and for the ſame reaſon the △ FAB = half □ FZ. Conſeq.

4 The □ upon AB (or AB^2) = the □ FZ; in the ſame manner may the □ BX made upon BC, be proved = the □ ZR, or that the □ ZR = BC^2: *for,*

if

if lines be drawn from A *to* X, *and from* B *to* R, *the argument will be the ſame.* But,

6 The ſquare made upon A C, (or AC^2,) is = the two □s F Z and Z R. (*per ax.* 9.) ∴

7 $AC^2 = AB^2 + BC^2$. *Q. E. D.*

For another more elegant demonſtration of this moſt uſeful theorem, ſee theo. 44.

Corollary.

HENCE, not only □s, but any other figures whatſoever, deſcribed upon the hypotheneuſe of a right angled △, will in area be equal to the areas of two ſimilar figures, deſcribed on the other two ſides taken together: *for, let the figures conſtructed be* △*s*, □*s*, *polygons, trapeziums, &c. as they may be reduced to* □*s*, (*per.* 207, 208, 209, and 210.) *therefore, the truth muſt be manifeſt.* But, for a further proof of this cor. *ſee cor.* 1 *to theo.* 43.

Note, from this theorem may any of the ſides of a ∟'d △ be found, if the other two be known; *e. g.* if the *baſe* and ⊥ be given, the *hypotheneuſe* may be thus found; add the areas of the □s on the given ſides together, and the *ſquare-root* * of their ſum will be the *hypotheneuſe* required.

Again, if the *hypotheneuſe* and either of the ſides be given to find the other ſide, you muſt ſubtract the area of the □ on the given ſide, from the area of the □ on the *hypotheneuſe*, and the *ſquare-root* of the difference will be the other ſide required.

The nine following theorems are taken from the ſecond book of Euclid, viz. the firſt ſix are regularly as they follow in Euclid, the remaining three

* By the term *ſquare root* here mentioned, you muſt underſtand it is a number, which, if multiplied by itſelf, the product will equal the ſquare of which it is the root; thus, 8 is the ſquare root of 64, for 8 × 8 = 64; again, 9 is the ſquare root of 81, for 9 × 9 = 81, &c. &c. And here obſerve, the numbers 64 and 81 are ſaid to be ſquare numbers. *See rem. to def.* 117, *where the ſquare is further explained.*

are

are the 11th, 12th, *and* 13th, *each of which (except the firſt) is demonſtrated two different ways, viz.* Firſt, *by reference,* i. e. *having a reference to the figure in the plate.* Secondly, *(for the advantage of the* Students *of* Trinity College) *in* general terms, *i. e. without having reference to any figure. In theſe theorems, I have been careful to demonſtrate them as nearly after* Tacquet's *method as poſſible, becauſe, that valuable book is not now to be met with; and, for the advantage of others who are not ſo particular, I have,* thirdly, *annexed a demonſtration of the firſt ſix theorems in* numbers, *whereby they will be rendered clear; and before we proceed, it will be neceſſary to premiſe the following particulars, or axoims:*

I. That any rgle may be expreſſed from the letters connected with the two lines, with which it is formed; as, if any line, as AB, be divided in C, (fig. 18) the rgle under its parts, viz. AB and CB, will be the rgle ACB, and the rgle under the whole line AB, and greater part AC, will be the rgle BAC; in all which expreſſions, the middle letter is ſuppoſed to be twice taken, as, the rgle in the firſt inſtance is a rgle under the right lines AC and CB multiplied together, *i e.* AC × CB, &c.

II. Where any right line is *equally* divided, the rgles under the ſegments will be □s; and, if *unequally* divided, the rgle under the whole line and greater part, will be greater than the rgle under the whole line and leſſer part; *i. e.* the rgle BAC, will be greater than the rgle ABC. fig. 18.

III. That all the rgles and □s under = right lines are =.

IV. That any letters connected with a line over them thus, $\overline{AC \times CB}$, ſignifies alſo the rgle ACB; and $\overline{AC^2 + CB^2}$, ſignifies the ſum of the ſquares of AC and CB taken together as one quantity.

Theo.

THEO. 17. 1 EUC. LIB. 2.

If there be taken two lines, one of them, as AC, *divided into as many parts as you pleaſe, viz.* AE, EF *and* FC, *and the other, as* AB; *undivided, the rgle* BAC *comprehended under them both, i. e.* BA × AC, *will be* = *to all the rgles under the undivided line* AB, *and the ſeveral parts of the divided line* AC.

fig. 17.

HYPOTHESIS.

That AB *is an* undivided *line and* AC *is a line* divided *into the parts* AE, EF *and* FC.

THESIS.

Rgle $\overline{BA \times AC}$ = *Rgle* $\overline{AE \times AB}$ + *Rgle* $\overline{EF \times AB}$ + *Rgle* $\overline{FC \times AB}$.

DEMONSTRATION.

The proof of this theorem depends chiefly, if not entirely, upon the conſtruction, which is as follows; on the line AC conſtruct the □ AI, the ſides of which, AB and AI, ſhall be reſpectively = to the given lines AC and AB (157) thus, on the points A, E, F, and C, (*the diviſions of the line* AC) erect the ⊥s AB, ED, FG and CI, (146 caſe II.) each = to the undivided line AB, and draw the line BI; now, as this □ AI contains exactly within it the ſeveral □s AD, EG and FI, each under the undivided line AB, and the ſeveral parts AF, EF and FC of the divided line, ∴ it muſt be = to them all together. (*per ax.* 9.) *Q. E. D.*

Or thus in Numbers.

LET the undivided line AB = 6 feet or any other meaſure; and the divided line AC = 12, be divided into the parts 2, 4 and 6. Then,

6 × 12 = 72 the rgle under BA and AC. Alſo,

6 × 2 = 12
6 × 4 = 24
6 × 6 = 36
} Are the rgles under the undivided line BA = 6, and the ſeveral parts of the divided line AC, viz. 2, 4 and 6. But,

4 The rgle 12 + rgle 24 + rgle 36 = rgle 72. ∴ &c.

THEO.

THEO. 18. 2 EUC. LIB. 2.

If a right line, as AB, *be divided any where, as in* C, *the* □ *of the whole line,* i e. AB^2 *will* = *two rgles under the whole line* AB, *and the parts* AC *and* CB. fig. 18.

HYPOTHESIS.	THESIS.
That the right line AB* *is divided into the two parts* AC *and* CB.	AB^2 = *rgle* ABC + *rgle* BAC.

DEMONSTRATION. 1 *by Reference.*

To prove this, let an undivided line, as F, be taken = AB. Then,

1 The rgle ABF, *i. e.* $\overline{AB} \times F$ = the rgle, ACF + the rgle CBF. (*Per theo.* 17, *and* 1 *E. lib.* 2). And,

2 The line F = AB, by conſtruction, conſeq the rgle ABF muſt be a □, and = to the □ of the given line. But,

3 The rgles ACF and CBF (in the firſt ſtep) are = to the rgles BAC, and ABC, *i.e.* to two rgles under the given line AB, and its parts AC and CB. ∴

4 The rgle ABF, *i. e.* AB^2 = the rgle ABC + the rgle BAC. *Q. E. D.*

DEMONSTRATION. 2 *in general Terms.*

Aſſume a line = to the given line, which, let be undivided; then the rgles under the ſeveral parts of the divided line, and the aſſumed line (*i. e.* the given line) will = the rgle under the given line and aſſumed line, (*per theo.* 17, *and* 1 *E. lib.* 2.) but the aſſumed line is = to the given line. ∴

* The letters made uſe of in the nine following theorems, are the ſame as thoſe uſed in *Whiſton's Euclid*, on purpoſe to anſwer the convenience of the Students in the College,

that

that rgle muſt be a □, and = to the two rgles under the given line and its parts. *Q. E. D.*

DEMONSTRATION. *3 in Numbers.*

Let the whole line AB = 12, be divided into the parts AC = 8, and CB = 4. Then,

The rgle BAC (*i.e.* $\overline{BA \times AC}$ = $\overline{12 \times 8}$ or 96. And,

The rgle ABC (*i.e.* $\overline{AB \times BC}$) = $\overline{12 \times 4}$ or 48. But,

96 + 48 = 144, *i. e.* = 12 × 12, (or, according to the theorem) the □ of 12 = the two rgles 96, and 48. *Q. E. D.*

Otherwiſe thus.

Let the line AB = 11, be divided into the parts AC = 7.5, and CB = 3.5. Then,

The rgle BAC = $\overline{11 \times 7.5}$ or 82.5. And,

The rgle BAC = 11 × 3.5 or 38.5 But,

The rgles 82.5, and 38.5 = 121, *i. e.* to AB^2, or $\overline{11 \times 11}$. *Q. E. D.*

It is particularly recommended to the young Student, to make as many diviſions of the given line as he can, and fractional diviſions in particular; for, if he be acquainted with decimals, ſuch trials will be of infinite ſervice to him; as, it will not only exerciſe him therein, but confirm him in the knowledge of theſe moſt uſeful theorems.

THEO. 19. 3 EUC. LIB. 2.

If a right line, as AB, *be divided any where, as in* C, *the rgle under the whole line* AB, *and either part, as* CB, *will be* = *to the rgle under the parts,* (*viz.* $\overline{AC \times CB}$) *added to the* □ *of the other part* CB. fig. 19.

HYPO-

HYPOTHESIS. THESIS.

That the line AB *is divided into the two parts* AC *and* CB. *The rgle* ABC = *the rgle* ACB + CB^2.

DEMONSTRATION. 1 *by Reference.*

To prove this, let a line, as F, be taken = to the part CB of the given line, which, let be undivided. Then,

1 The rgle ABF (*i. e.* the rgle ABC) is equal the rgle ACF + the rgle CBF. (*per theo.* 7, *and* 17 *E. lib.* 2.) But,

2 The line F = CB (*per preparation*) conseq the rgle ACF = the rgle ACB, and the rgle CBF = BC^2. ∴

3 The rgle ABF (*i. e.* the rgle ABC) = the rgle ACB + CB^2. *Q. E. D.*

DEMONSTRATION. 2 *in general Terms.*

Assume a line = one part, then the rgle under the the whole line and assumed line, will = two rgles under the parts and assumed line. (*per theo.* 17, *and* 1 *E. lib.* 2.) But, as the assumed line was taken = one of the parts, conseq the rgle under the assumed line and its = part, will be the □ of that part, and the other rgle is under the parts. ∴ &c. *Q. E. D.*

DEMONSTRATION. 3 *in Numbers.*

Let the whole line AB = 12, be divided into the parts AC = 7, and CB = 5. Then,

The rgle ACB or $\overline{AC \times CB} = 7 \times 5 = 35$. And
The □ of BC = $5 \times 5 = 25$. But,
$35 + 25 = 60$, *i. e.* to $\overline{12 \times 5}$. *Or according to the theorem, the rgle* $\overline{12 \times 5}$ = *the rgle* $\overline{7 \times 5} + \overline{5 \times 5}$ (*i. e.* the □ of 5). *Q. E. D.*

THEO.

THEO. 20. 4. EUC. LIB. 2.

If a right line, as FL, *be divided any where, as in* O, *the* □ *of the whole line will be* = *to two rgles, under the parts* + *two* □*s of the parts.* fig. 20.

HYPOTHESIS.

The line FL *is divided into two parts,* FO *and* OL.

THESIS.

FL^2 = *twice the rgle* $FOL + FO^2 + OL^2$.

DEMONSTRATION. 1 *by Reference.*

If the line FL was equally divided, the theorem muſt be manifeſt; for, the rgles under the parts muſt be □s, they being =, and the other two rgles, would be □s of the parts, and all together = the □ of FL. (*per ax.* 9.) ∴ &c. But, if the line be not equally divided, then,

1 The rgle FLO = the rgle $FOL + OL^2$. (*per theo.* 19 *and* 3 *E. lib.* 2). Alſo,

2 The rgle LFO = the rgle $FOL + FO^2$. (*per the ſame*). But,

3 The rgle FLO + the rgle LFO = FL^2, becauſe it is FL (*i. e.* the whole line) multiplying its parts, viz. FO and OL, or, the line multiplying itſelf. ∴

4 FL^2 will = twice the rgle $FOL + FO^2 + OL^2$ (*per* 1 *and* 2). *Q. E. D.*

DEMONSTRATION. 2 *in general Terms.*

The rgle under the whole line, and one part, will be = to a rgle under the parts; + the □ of that part (*per theo.* 19. *and* 3 *E. lib.* 2.) in the ſame manner the rgle under the whole line, and the other part, will be = to a rgle under the parts + the □ of the other part; but theſe two rgles toge-

 ther,

ther, will be = to the □ of the whole line; ∴ this □ will be = to two rgles under the parts + two □s of the parts. *Q. E. D.*

DEMONSTRATION. *3 in Numbers.*

Let the line FL = 12, be divided into the parts FO = 8, and OL = 4. Then,

The rgle FOL or $\overline{FO \times OL} = 8 \times 4 = 32$. And,

The rgle LOF or $\overline{LO \times OF} = 4 \times 8 = 32$. Alſo,

The □s FO & LO *i. e.* $8 \times 8 \& 4 \times 4 = 80$. Which □s and rgles added will = $\overline{144}$, that is, to the ſquare of the whole line, viz. 12×12. ∴ &c.

COROLLARY 1.

The ▭s about the diagonal of a □ are □s. For *the lines which form theſe ▭s, being ‖ to the ſides of the □, will produce = ∠s with the □. (per. theo. 8, and 27 E. lib. 1) i. e. ∟s, and intercept = parts of the oppoſite ſides of the □ (per. def. 39.) ∴ they will be both equiangular and equilateral, conſeq. □s.*

COR. 2. The diagonal of every □ biſects its ∠'s. *For, in the △s into which the □ is divided, there are two ſides = and the contained ∠ a right one. ∴ the ∠s at the baſe muſt be half right ones. (per cor. 5, theo. 9, and cor. 11. 32 E. lib. 1.)*

COR. 3. The □ of half a line will = the fourth part of the □ of the whole line. *For, the □ of the whole line will = the two □s of its parts + two rgles under its parts (by theo. 20, and 4 E. lib. 2.) but theſe rgles and □s being under = parts, (or half the line) muſt be □s and = each other ; ∴ one of theſe □s muſt be a fourth part of the whole line.*

COR. 4. Every □ is = to twice the rgle contained under one of its ſides, and half the other ſide. *For, the rgles into which the □ is divided are =, but, the □ is = to them both together, conſeq muſt be double of either.* Alſo every ▭ is = to twice an equi-angled ▭ under any one of its ſides, and half the other.

THEO-

THEO. 21. 5 EUC. LIB. 2.

If a right line, as QX, *be equally divided in* R, *and unequally in* S, *the rgle under the unequal parts* QS *and* SX, *together with the* □ *of the intermediate part* RS, *will* = *the* □ *of half the line.* fig. 21.

HYPOTHESIS.
The line QX *is divided equally in* R, *and unequally in* S.

THESIS.
The rgle QSX + RS^2 *will* = RX^2.

DEMONSTRATION. 1 *by Reference.*

1 The rgle QSX = the rgle $\overline{QR} \times \overline{SX}$ + the rgle RSX. (*per theo.* 17. *and* 1 *E. lib.* 2.) And,

2 As QR = RX) the rgle $QR \times SX$ (*i. e.* the rgle RXS) will = the rgle $RSX + SX^2$. (*per theo.* 19. 3 *E. lib.* 2). That is,

3 The rgle QSX will = twice the rgle $RSX + SX^2$, (*per* 1 *and* 2) now, if to both sides of this equation you add RS^2. (*i. e. the* □ *of the intermediate part.*) Then,

4 The rgle $QSX + RS^2$, will = twice the rgle $RSX + SX^2 + RS^2$. (*per ax.* 2). But,

5 Twice the rgle $RSX + SX^2 + RS^2$ will = RX^2. (*per theo.* 20. *and* 4 *E. lib.* 2). ∴

6 The rgle $QSX + RS^2$ will = RX^2 (*by comparing the arguments in the* 4*th and* 5*th steps*) *i. e.* to the □ of half the given line. *Q. E. D.*

DEMONSTRATION, 2 *in general Terms.*

The rgle under the unequal parts, will = a rgle under the half line and leſſer part, + the rgle under the intermediate and leſſer parts. (*per theo.* 17. *and* 1 *E. lib.* 2). Again, the rgle under the half line and leſſer part, will = the rgle under the intermediate and leſſer part, + to the □ of the leſſer part;

(*per*

(*per theo.* 19. *and* 3 *E. lib.* 2.) wherefore, the rgle under the unequal parts, will = two rgles under the intermediate and leſſer + to the □ of the leſſer. Now, if to theſe =s you add the □ of the intermediate part, then the rgle under the unequal parts, + the □ of the intermediate part, will = two rgles under the intermediate and leſſer, + the □s of the intermediate and leſſer; but, theſe two rgles and □s are = to the □ of half the given line (*per theo.* 20. *and* 4 *E. lib.* 2). ∴ &c. *Q. E. D.*

DEMONSTRATION. 3 *in Numbers.*

Let the whole line QX = 12, then QR and RX will each = 6, and let RS = 2 and SX = 4, then QR + RS (or QS) will = 8. Then,

The rgle QS X or 8 × 4, will = 32. Again,
The □ of RS or 2 × 2, will = 4, which
rgle and □ added together, will = 36, that is, = to the □ of QR or RX, or to 6 × 6, the □ of half the line. *Q. E. D.*

COROLLARY 1.

The rgle under the unequal parts of any line, unequally divided, is *leſs* than the □ of half the line. For, *the rgle under the unequal parts + the □ of the intermediate part is = the □ of the half line* (per theo. 21, and 5 E. lib. 2.) ∴ &c.

COR. 2. The nearer the point of unequal diviſion approaches the central point, the greater will be the rgle under the unequal parts, and the contrary. *For, by approaching it will leſſen, and by receding encreaſe, the □ of the intermediate part which is always the compliment of the rgle of the unequal parts to the □ of half the line.*

COR. 3. The nearer the point of unequal diviſion approaches the central point, the leſs will be the ſum of the □s of the unequal parts; and the further it recedes from it, the greater will be that ſum. *For, the □ of the whole line is = to two □s of the unequal parts + two rgles under*

under theſe parts. (per tho. 20, and 4 E. lib. 2.) *But, the point of unequal diviſion, by approaching the central point, will enlarge, and by receding will diminiſh the rgle under the unequal parts,* (per the foregoing cor.) *∴ muſt accordiugly* enlarge *or* diminiſh *the ſum of the* □*s of thoſe parts.*

Cor. 4. The rgle under the unequal parts taken four times, will be leſs than the □ of the whole line. *For four times the* □ *of half the line is* = *to the* □ *of the whole line* (per cor. 3, theo. 20, and 4 E. lib. 2.) *But, the rgle under the unequal parts is leſs than the* □ *of half the line, by the* □ *of the intermediate part* (per cor. 1, of this) *conſeq if taken four times, it muſt be leſs than the* □ *of the whole line, by four times the* □ *of the intermediate part.*

Cor. 5. If two equal right lines be ſo divided, that the rgles under the parts of each line be =, then will the parts of one be = to thoſe of the other. *For, if the right lines be biſected the cor. will be manifeſt; but, if not, let them be biſected, and their halves will be* =, *conſeq the* □*s of thoſe halves will be* = *alſo; but thoſe* □*s are* = *to the rgles under the unequal parts* + *the* □*s of the intermediate parts; now, let theſe* = *rgles be taken away from both theſe* □*s, and the* □ *of the intermediate part of one line will remain* = *to the* □ *of the iutermediate part of the other, and as theſe* □*s are* = *alſo the roots or ſides of theſe* □*s muſt be* =, *which ſides let be added to the* = *halves on one ſide, and taken from the* = *halves on the other, and the parts of the two right lines will therefore be* =.

Cor. 6. The rgle under the ſum and difference of two unequal right lines, will be = to the difference of the □s made by thoſe two right lines. *For, let them be laid in one line, which, let be produced until the whole line be* = *twice the greater. Then the rgle under the ſum and difference, together with the* □ *of the leſſer, will be* = *to the* □ *of the greater.* (per theo. 21, and 5 E. lib. 2.) i. e. *the rgle under the ſum and difference, will be* = *the difference between thoſe* □*s.*

Theo. 22. 6 Euc. Lib. 2.

If a right, line as AB, *be equally divided in* C, *and to it a rigbt line, as* BF, *be added, the rgle under the whole compound line* AF, *and the*

the part added BF, *together with the* □ *of half the line* CB, *will be* = *to the* □ *of* CF; *that is, to a line compounded of the half line and part added.* fig. 22. No. 1.

HYPOTHESIS.	THESIS.
The right line AB *is biſected in* C, *and the right line* BF *is added to it.*	*The rgle* AFB + CB^2 *will* = CF^2.

DEMONSTRATION. 1 *by Reference.*

To prove this, let another line LA be taken = to BF, and added to the other extremity of AB. Then,

1 LB = AF. (*per ax.* 2) And,

2 The rgle LBF + CB^2, will = CF^2. (*per theo.* 21. *and* 5 *E. lib.* 2). But,

3 The rgle LBF = the rgle AFB, becauſe, LB = AF. (*per* 1 *ſtep*). ∴

4 The rgle AFB + CB^2 = CF^2. (*per* 2 *ſtep*). *Q. E. D.*

Otherwiſe thus, without the addition of LA. fig. 22. No. 2.

1 The rgle AFB = the rgle CBF + the rgle $\overline{AC \times BF}$ (*i. e.* the rgle CBF, for AC = BF) + BF^2 (*per theo.* 17 *and* 1 *Euc. lib.* 2.) that is, the rgle AFB = twice the rgle CBF + BF^2, *by ſubſtituting the* = *rgles.* To this add CB^2. Then,

2 The rgle AFB + CB^2 = twice the rgle CBF + BF^2 + CB^2. (*per ax.* 2). But,

3 Twice the rgle CBF + BF^2 + CB^2 = CF^2. (*per theo.* 20, *and* 4 *E. lib* 2.) ∴

4 The rgle AFB + CB^2 = CF^2. *Q. E. D.*

DEMON-

DEMONSTRATION. *2 in general Terms.*

The rgle under the whole compound line, and part added, will = two rgles under half line, and part added, + the □ of the part added, (*per theo.* 17, & 1 *E. lib.* 2.) to which equals let the □ of half line be added; then, the rgle under the whole compound line, and part added, + the □ of half line, will = two rgles under half line, and part added, + □ of part added, + the □ of half line (*per ax.* 2); but, these two rgles and □s are = to the □ of a line compounded of half line, and part added (*by theo* 20 & 4 *Euc. lib.* 2) ∴ the rgle under the whole compound line, and part added + the the □ of half line, will = the □ of a line compounded of half line, and part added. *Q. E. D.*

DEMONSTRATION. *3 in Numbers.*

Let the line A B = 12, then A C and C B, its half, will each = 6, and let B F = 4 be added, consequ. the compound line A F will = 16. Then,

The rgle A F B, *i. e.* A F × F B or 16 × 4 = 64. And,
The □ of C B *i. e.* 6 × 6 = 36, which
rgle and □, viz. 64 and 36 added = 100, *i. e.*
to the □ of C F, or 10 × 10. *Q. E. D.*

COROLLARY.

If three right lines be arithmetically proportional (*see def.* 120.) the rgle under the extremes (i. e. the first and last) together with the □ of the difference, will be = to the □ of the mean. *For, let the lesser extreme and middle term be severally laid off on the greater extreme, in which case you will have the difference between the two extremes, bisected by the extremity of the mean* (because the differences between the terms are =) *to this bisected line the lesser extreme will be added; wherefore by this proposition the rgle under the whole compound line and the adjoining one* (i. e. the extremes) *together with the* □ *of half the line* (i. e. the difference) *will be = to the* □ *of a line, compounded of the half line and adjoining one* (i. e. the □ of the mean.)

THEO.

THEO. 23. 7 EUC. LIB. 2.

If a right line, as A B, *be divided any where in* C, *the* □ *of the whole line, together with the* □ *of one of the parts, as* AC, *will* = *twice the rgle* B A C *under the whole line, and that part together with the* □ *of the other part* C B. fig. 23.

HYPOTHESIS. THESIS.

The right line A B, *is equally divided in* C. $AB^2 + AC^2 =$ *twice the rgle* $BAC + BC^2$.

DEMONSTRATION. 1*ſt by Reference.*

1 $AB^2 =$ twice rgle, $BCA + BC^2 + AC^2$, (*per theo.* 20 *and* 4 E. *lib.* 2.) And,

2 The rgle B A C = the rgle $BCA + AC^2$, (*per theo.* 19, *and* 3 E. *lib.* 2.) Again,

3 Twice the rgle B A C = twice the rgle BCA + twice AC^2, (*per ax.* 6.) now, if from the firſt ſtep you take the third, it will be

4 AB^2 — twice B A C will = $BC^2 - AC^2$, that is,

5 $AB^2 + AC^2 =$ twice rgle B A C + BC^2 *by adding the* □ AC *to one ſide of the equation,* and the *double rgle* B A C *to the other* (*per ax.* 2.) *Q. E. D.*

DEMONSTRATION. 2 *in general terms.*

The □ of the whole line will = two rgles under the parts, + two □s of the parts, (*per theo.* 20 *and* 4 E. *lib.* 2.) and, the rgle under the whole line and one part, will = a rgle under the parts + □ of that part, (*per theo.* 19, *and* 3 E. *lib.* 2.) this laſt equation being doubled, and taken from the firſt, will leave the □ of the whole line, — twice the rgle under the whole line and one part, = to the □ of one part, — the □ of the other part; then, let the □ of the other part

part be added to one ſide of this equation, and the double rgle to the other; and the □ of the whole line, + the □ of one part, will = twice the rgle under the whole line, and that part + □ of the other part. *Q. E. D.*

COROLLARY.

If from a right line one part be cut off, the □ of the remainder, is exceeded by the □s of the whole line, and part cut off, taken together, by two rgles contained under the whole line and part cut off. *For the □ of the whole line + the □ of the part cut off = two rgles under the whole line, and part cut off + the □ of the other part* (per this theo. and 7 E. lib. 2.) *Now, by taking from both ſides the two rgles under the whole line and part cut off, then will the □ of the whole line + the □ of the part cut off — two rgles under thoſe lines = the □ of the other part.*

THEO. 24. 11 EUC. LIB. 2.

To cut a line, as AB, *in* C, *ſo that the □ of one part, as* AC, *ſhall be = to a rgle under the whole line* AB, *and the other part* CB, or, to cut it into extreme and mean proportion. fig. 24.

GIVEN. SOUGHT.

The right line AB. *The point* C, *ſo that the rgle* ABC *or* AB × BC *ſhall* = AC^2.

To find the point C, proceed thus:

1 From the point A, draw the ⊥ AD, (146 caſe II) which make = to AB. 2. Biſect AD in I, (144) and draw the line IB. 3 Continue the line DA to G, ſo that IG ſhall be = to IB. 4 Take AG

and transfer it to the line AB, and it will cut it in C, the point required.

DEMONSTRATION. 1 *by Reference.*

1 The rgle DGA + $AI^2 = IG^2$, (*per theo.* 22, *and* 6 *E. lib.* 2.) *i.e.* to IB^2, (for IB = IG). Again,

2 $IB^2 = AI^2 + AB^2$ (*per theo.* 16, *and* 47 E. *lib.* 1.) *i. e. the rgle* $DGA + AI^2 = AB^2 + AI^2$ (*per ax.* 2) *here* AI^2 *is common to both ſides of this equation, wherefore, taking it away there will remain,*

3 The rgle DGA = AB^2, (*per ax.* 3). But,

4 The rgle DGA = the rgle DAG + AG^2, (*per theo.* 19, *and* 3 *E. lib.* 2.) *i. e.* to the rgle BAC + AC^2, (*for* BA *and* AC = DA *and* AG, *per conſtruction*). But alſo, obſerve

5 AB^2 = the rgle BAC + the rgle ABC, (*per theo.* 18, *and* 2 *E. lib* 2). Conſeq,

6 The rgle BAC + AC^2, = the rgle BAC + the rgle ABC (*per ax.* 1, *and* 3*d*, 4*th*, *and* 5*th*, *ſteps*) but, *here the rgle* BAC, *is common to both ſides of this equation; wherefore, taking it away, there will remain*

7 The rgle ABC = AC^2. *Q. E. D.*

DEMONSTRATION. 2 *in general Terms.*

1 Let it be obſerved, that if the difference (*between a line ſubtending any given line, and its half, placed at* ∟*s, and half the given line)* be cut from the given line, it will divide it as required; from which expreſſion of the problem in terms, take the following demonſtration.

The rgle under a line, compounded of that ſubtence and half line together, into the difference + the □ of half the line, will = the □ of that ſubtence, (*per theo.* 22, *and* 6 *E. lib.* 2.) *i. e.* to the ſum of the □s of the line and half line; (the line and half line being at ∟s, *per theo.* 16, *and* 47 *E. lib.* 1) here the □ of the half line is common to both this rgle and □, wherefore, by taking it away, this

rgle

rgle will = the □ of the whole line; but this rgle will alſo = the rgle under the given line, and difference + □ of the difference, (*per theo.* 19, *and* 3 *E. lib.* 2) and, the □ of the given line, will = a rgle under the given line and difference, *i. e.* one part, + a rgle under the given line and the other part, (*per theo.* 18, *and* 2 *E. lib.* 2) conſeq, if you take away the rgle under given line and difference, from this rgle and □, (*it being a common rgle,*) the remainder, viz. the rgle under the given line and one part, will = the □ of the difference, but the difference = the other part of the line, ∴ the rgle, &c. *Q. E. D.*

Note, this theorem cannot be exemplified in numbers, becauſe no number whatſoever can be ſo divided, that the rgle under the whole number and one part, ſhall be = the □ of the other part; and, notwithſtanding the impoſſibility of performing this in numbers, yet, the force of this ſection of a line geometrically is wonderful.

THEO. 24. 12 EUC. LIB. 2.

In every obtuſe ∠'d △, as ACB, *the* □ *of the ſide* AC *ſubtending the obtuſe ∠* B, *is greater than the ſum of the* □*s of the other two ſides* AB *and* CB, *by twice the rgle* CBF, *which rgle is comprehended under* BC, *one of the ſides of the obtuſe ∠, and the line* BF, *intercepted between the ⊥* AF *and the obtuſe ∠.* fig. 25.

HYPOTHESIS.

The figure ABC *is a* △ *obtuſe ∠'d at* B, *and* AF *is a ⊥ let fall upon* CB *produced.*

THESIS.

AC^2 *is greater than* $AB^2 + CB^2$, *by twice the rgle* CBF, *or* $AC^2 = AB^2 + 2$ CBF.

DEMON-

DEMONSTRATION. 1 *by Reference.*

1 $AC^2 = CF^2 + AF^2$ (*per theo.* 16, *and* 47 *E. lib.* 1). Again,

2 $CF^2 = FB^2 + CB^2 +$ twice the rgle CBF (*per theo.* 20, *and* 4 *E. lib.* 2.) *i. e.* $AC^2 = AF^2 + FB^2 + CB^2 +$ twice the rgle CBF (*per ax.* 1). But,

3 $AF^2 + BF^2 = AB^2$ (*per theo.* 16, *and* 47 *E. lib.* 1). ∴

4 $AC^2 = AB^2 + CB^2 +$ twice the rgle CBF, *i. e.* AC^2 is greater than $AB^2 + CB^2$, by twice the rgle CBF. *Q. E. D.*

DEMONSTRATION. 2 *in general Terms.*

The □ of the ſide ſubtending the obtuſe ∠, will = the □ of a ⊥, let fall from this ſubtence, to the oppoſite ſide of the △ produced, together with the □ of a line compounded of the produced ſide and the intercepted part (*per theo.* 16, *and* 47 *E. lib.* 1.) Alſo, the □ of this compound line will = two rgles under the produced line and intercepted part, + two □s of the produced line, and the intercepted part, (*per theo.* 20, *amd* 4 *E. lib.* 2) but the □ of the other ſide of the obtuſe ∠ will = the □ of the ⊥ + the □ of the intercepted part, *i. e.* the □ of the ſide oppoſite the obtuce ∠ will = the ſum of the □s of the other two ſides, + two rgles under the produced line and intercepted part, ∴ the □ of the ſide oppoſite the obtuce ∠, is greater than the □ of the other ſides, by twice this rgle, *Q. E. D.*

THEO. 26. 13 EUC. LIB. 2.

In any △ *whatſoever, as* ABC, *the* □ *of the ſide* AB *oppoſite to any acute* ∠, *as* C, *is leſs than the ſum of the* □*s of the other two ſides* AC CB, (*about the acute* ∠), *by twice the rgle* BCF, *contained under* BC, *one of thoſe ſides of the acute* ∠, *and that part* FC *of the ſame*

ſame, intercepted between the acute ∠ C, and a ⊥ let fall upon that ſide from the oppoſite ∠ A. fig. 26.

HYPOTHESIS.

That ABC *is a* Δ, *and* AF *a* ⊥ *let fall from the* ∠ A *upon the oppoſite ſide* BC, *meeting it in* F, *either within or without the* Δ.

THESIS.

BA^2 *oppoſite the accute* ∠ C *is leſs than* $BC^2 + AC^2$ *by twice the rgle* BCF, *or* $BC^2 + AC^2 =$ *twice rgle* BCF + AB^2.

CASE I. Let the Δ ABC be any oblique ∠d Δ, conſeq the ⊥ AF will fall within the Δ. fig. 25 or 26.

DEMONSTRATION. 1 *by Reference.*

1 $BC^2 + CF^2$ = twice the rgle BCF + BF^2, (*per theo.* 23, *and* 7 *E. Lib.* 2) to which add AF^2. Then,

2 $BC^2 + CF^2 + AF^2$ = twice BCF + $BF^2 + AF^2$. (*per. ax.* 2.) But,

3 $CF^2 + AF^2 = AC^2$, alſo, $BF^2 + AF^2 = AB^2$. (*per theo.* 16, *and* 47 *E. lib.* 1.) Conſeq

4 $BC^2 + AC^2$ = twice rgle BCF + AB^2. *Q. E. D.* or AB^2 is leſs than $BC^2 + AC^2$ by twice rgle BCF. *Q. E. D.*

CASE II. Suppoſe the Δ to be a ∟'d Δ, as AFB, in figs. 25 or 26, then BF will be the right line between the acute ∠ B and the ⊥ AF.

DEMONSTRATION.

1 $AB^2 = AF^2 + BF$ (*per theo.* 16 *and* 47 *E. Lib.* 1) to which add BF^2. Then,

2 $AB^2 + BF^2 = AF^2 + 2\,BF^2$, (*per ax.* 2.) ∴

3 AF^2 is leſs than the $AB^2 + BF^2$ by $2\,BF^2$. Conſequently in any Δ whatever, the theorem is equally true.

DEMON-

DEMONSTRATION. *2 in general Terms.*

The □ of the ſide on which the ⊥ falls *(produced if neceſſary)* + the □ of line between the acute ∠ and ⊥ will = two rgles under that ſide, and that line + the □ of their difference *(per theo. 23 & 7 E. lib. 2.)* To both theſe =s, add the □ of the ⊥; then the □ of the ſide on which the ⊥ falls + the □ of the line between the acute ∠ and ⊥, + the □ of the ⊥, will = two rgles under the aforeſaid ſide and line + the □ of the ⊥ *(per. ax 2.)* But, the □ of the line between the acute ∠ and ⊥, + the □ of the ⊥, will = the □ of the other ſide containing the acute ∠, and alſo the □ of the aforeſaid difference + the □ of the ⊥, will = the □ of the ſide oppoſite the acute ∠ *(per. theo. 16. & 47 E. lib. 1.)* ∴ by ſubſtituting the values of the □s of the ſides about the acute ∠, in the place of their =s; then, will the □ of the ſide oppoſite the acute ∠, be leſs than the ſum of the □s of the ſides about the acute ∠, by twice the rgle contained under that ſide on which the ⊥ falls and the line between the acute ∠ and ⊥, as per theo.

Scholium. From the above theorem may the altitude of any △ *be found, from having the three ſides known, and conſequently from the 16th theorem the area thus, in the* △s fig. 25 *and* 26, *it will be by this theo.* the □ of the ſide oppoſite the acute ∠ is leſs than the ſum of the □s of the two ſides about the acute ∠ by twice the rgle contained under that ſide on which the ⊥ falls, and the line between the acute ∠, and a ⊥ let fall from the oppoſite ∠, i. e. AB^2 *is leſs than* $AC^2 + CB^2$ *by twice the rgle* BCF, *wherefore, let* AB^2 *be taken from* AC^2 *and* CB^2, *and there will remain twice the rgle* BCF, *the half of which will = the rgle* BCF *only; now, let this rgle* BCF *be divided by the given line* BC, *and the quotient will = the line* CF*. *Wherefore we*

* For the rgle BCF is BC × CF; *i. e.* BCF is the product of BC and CF, *(ſee note to def. 52)* conſeq. the product divided by either factor as BC, the quotient will = the factor, *i. e.* CF; as muſt be evident from common diviſion.

ſhall

ſhall have in the ∟*'d* △ CAF, *the two ſides* AC *and* CF *to find* AF *the* ⊥ *required, which,* (by the 16th theo. and 47 Euc. Lib. 1) *it will be,* $CA^2 - CF^2 = AF^2$ *the ſquare root of which will be* AF, *conſeq.* $AF \times \frac{CB}{2}$ = *the area of the* △. (See prob. 64.) *Q. E. F.*

THEO. 27. 3 EUC. LIB. 3.

In any circle, as ADBE, *if a diameter* AB *or radius* CB *be drawn* ⊥ *to any chord, as* DE, *it will biſect both the chord and its ſubtended arch.* fig. 27.

HYPOTHESIS.	THESIS.
AB *is the diameter of the circle* ADBE, *and is* ⊥ *to the chord* DE.	1 DF = FE, *and* 2 *the arch* DB = *the arch* BE.

To prove this, let the radii CD and CE be drawn.

DEMONSTRATION.

1 In the △s DCF and ECF, the ſides DC and CE are =, *(per. remark to def.* 14 *and* 15) and the ſide CF is common to both, *i. e.* the ſides DC and CF in one △, is = to the ſides EC and CF in the other. But,

2 The ∠ DCF = the ∠ ECF, *(per cor.* 3, *to theo.* 9) For, the ∠s at F are ∟s, and the ∠ D = the ∠E, *(per theo.* 5) the △ DCE being an iſoſceles △. ∴

3 The line DF = the line FE *(per theo.* 1,) *Q. E. D. in the firſt part.* Again,

4 Becauſe in the two △s DCF and EFC, the ∠s at C are = (*per* 2 *ſtep.*) The arches DB and BE which meaſure theſe ∠s muſt be = alſo, for the quantity of degrees contained in each arch will be the ſame. *(See def.* 29.) *Q. E. D. in the ſecond part.*

COROLLARY

COROLLARY 1.

Hence, if a diameter or any radius of a circle biſect any chord, the chord will be cut perpendicularly. *For, the* △s DCF *and* ECF *are congruous to each other, wherefore, the* ∠s *at* F *are* = ; *conſeq* ∟s, ∴ CF *muſt be* ⊥. (3 E. lib. 3.)

COR. 2. In = circles = chords will ſubtend = arches, and the contrary. *For, notwithſtanding the* = *chords* DB *and* BE *are drawn in the ſame circle, yet, were they drawn in different but* = *circles, the corollary would be equally true.* (26 and 27 E. lib. 3.)

THO. 28. 16 EUC. LIB. 3.

Any right line, as AB, *touching a circle in the point* C, *the extremity of the diameter* FC, *it will be* ⊥ *to it, touch it in* C *and fall entirely without the circle; and no right line can be drawn between the tangent and the circle without cutting the circle.* fig. 28.

HYPOTHESIS.

The right line AB *is a tangent to the circle* CEFC, *and touches the extremity of the diameter* FC *in the point* C.

THESIS.

1 AB *is* ⊥ *to* CF *and touches the circle in* C.

2 *The line* AB *will fall entirely without the circle.*

3 *No right line can be drawn between the tangent and the circle to the point* C, *without cutting the circle.*

DEMONSTRATION.

1 If it be denied that CF is ⊥ to AB, let ſome other line as DG be drawn ⊥, but it will cut the circle in E; then becauſe the line DG is ⊥, the ∠ DGC will be a ∟, and the ∠ DCG acute, (*per cor.* 2, *theo.* 9) conſeq the ſide DC oppoſite the greater ∠, muſt be greater than DG oppoſite the leſſer ∠, (*per theo.* 13) *i.e.* the line DC (= DE) is greater than

than D G, or a part greater than a whole, which is abſurd, (*and contrary to ax.* 9) and as the ſame abſurdity will appear by drawing a line in any other direction than D C, ∴ A B muſt be ⊥ to C F. *Q. E. D. in the firſt part.*

2 From the conſtruction in the firſt part, ſeeing that D C reaches to the circumference, the line D G being greater, muſt reach beyond it, ∴ the point G the extremity of D G, which is in the line A B, muſt be outſide the circle; and in the ſame manner every other point in A B, may be proved to be without the circle. *Q. E. D. in the ſecond part.*

3 If any right line can poſſibly be drawn between the tangent A B and the circle to the point of contact C, without cutting it, let it be the line C H; draw the right line D I ⊥ to C H, (148) then, becauſe in the △ D I C the ∠ I is a ∟, the line D C oppoſite to it, muſt be greater than D I, which is oppoſite to the acute ∠ C, (*per part* 2, *theo.* 13.) *i. e.* the line D C (= D E) is greater than D I; now, perceiving the lines D C and D E reach to the circumference, the line D I which falls upon C H, cannot reach ſo far, conſeq the line C H muſt paſs through the circle and cut the circumference in E. ∴ &c. *Q. E. D. in the third part.*

Corollary 1.

Hence a right line and a circle touch each other in a point only. *For, every point in* A B *is entirely without the circle, but the point* C, *where it touches it.* 13 E. lib. 3.

Cor. 2. If any right line be drawn in a circle from the point of contact ⊥ to the tangent, it will divide the circle into two = parts or ſegments, and paſs through the center. *For, was the line* C F *drawn in any other direction than through the center, the line* A B *would not* (from the third part of this theo.) *be* ⊥ to it, ∴ &c. 19 E. lib. 3.

Cor. 3. Hence alſo if any right line touch a circle, and another right line drawn from the center to the point of contact, it will be ⊥ to the tangent. *This muſt be true from Cor.* 2. 18 E. lib. 3.

THEO. 29. 20 EUC. LIB. 3.

The ∠ BCD *at the center of a circle, is double to the* ∠ BAD *at the circumference, when they both ſtand on the ſame arch.* fig. 29.

HYPOTHESIS.

The ∠ BCD *is an* ∠ *at the center, and the* ∠ BAD *another at the circumference, and both* ∠*s on the ſame arch* DB.

THESIS.

∠ BCD *is double the* ∠ BAD; *that is, the* ∠ BCD = 2 ∠ BAD.

DEMONSTRATION.

In this theorem there are three caſes, according to the poſitions of the lines of the ∠*s.*

CASE I. When one leg of both ∠s coincide, as in fig. 29, No. 1.

Then the ∠ BCD = the ∠s A and D together being external to both, (*per theo.* 9.) But the ∠ A = ∠ D (*per theo.* 5.) ∴ the ∠ BCD muſt be double the ∠ A, *i. e.* to the ∠ BAD.

Q. E. D.

CASE II. When the legs of the ∠s do not coincide, and the ∠ at the center falls within the ∠ BAD at the circumference, as in fig. 29, No. 2.

In this caſe, from A, through C, draw the right line AE then the ∠ BCE will be double the ∠ BAE (*per caſe* I *of this*) and for the ſame reaſon, the ∠ ECD will be double the ∠ EAD; but the ∠ BCE + the ∠ ECD is = the ∠ BCD at the center, ∴ it is double the ∠ BAE + the ∠ EAD, *i. e.* the ∠ BAD at the circumference. *Q. E. D.*

CASE III. When the ∠ BCD at the center falls entirely without the ∠ BAD at the circumference, as in fig. 29, No. 3.

Here,

Here, through C, draw the line A E, as in caſe II; then will the whole ∠ E C D be double the whole ∠ E A D, (*per caſe* I *of this*) and in ſame manner will the ∠ E C B be double the ∠ E A B; let theſe latter ∠s be taken from the former, (*they being common*) and the remaining ∠ B C D, will be double the remaining ∠ B A D. ∴ &c. *Q. E. D.*

COROLLARY 1.

Hence any ∠ whatſoever, as B A D at the circumference of a circle, may be meaſured by half the arch which ſubtends it, or on which it ſtands. *For, the* ∠ B C D *at the center, is meaſured by the arch* B D *on which it ſtands* (per rem. def. 29) *conſeq the* ∠ B A D, *its half, muſt be meaſured by half the arch* B D.

COR. 2. All the ∠s in the circumference of any circle, ſtanding on the ſame arch (or which is the ſame thing) in the ſame ſegment, are =. *For, they are all meaſured by half the ſame arch.* 21 *E. lib.* 3.

COR. 3. An ∠ in a ſemi-circle is always a ∟. *For, as it ſtands upon a ſemi-circle, it is* (per cor. 1 of this) *meaſured by half that arch, which is a quadrant, or* 90°. (per def. 30, and its rem.) 31 E. lib. 3.

COR. 4. An ∠ in a ſegment greater than a ſemi-circle, will be leſs than a ∟. *For, it is meaſured by half the other ſegment, on which it ſtands, being leſs than a ſemi-circle,* ∴ *its half muſt be leſs than a quadrant or* 90°. Part 2, 31 E. lib. 3.

COR. 5. An ∠ in a ſegment leſs than a ſemi-circle, will be greater than a ∟. *For, it is meaſured by the other ſegment on which it ſtands, which, being greater than a ſemi-circle, its half muſt be greater than a quadrant or* 90°. Part 3, 31 E. lib. 3.

COR. 6. If in equal circles, the ∠s, whether at the circumference, or at the center, be =, the arches on which they ſtand muſt be = alſo; and the contrary. *This muſt be true from the above corollaries.* 29 E. lib. 3.

THEO.

THEO. 30. 22 EUC. LIB. 3.

The oppoſite ∠s of any quadrilateral, as ABCD *inſcribed in a circle, are together = to two ∟s.* fig. 30.

HYPOTHESIS. THESIS.

The figure ABCD *is a quadrilateral, inſcribed in a circle* ∠ A + ∠ C = 2 ∟s, *alſo* ∠ B + ∠ D = 2 ∟s.

To prove this, let the diagonal lines AC and BD be drawn.

DEMONSTRATION.

1 The ∠ BAD + ∠ *a* + ∠ *b* = 2 ∟s. (*per theo.* 9.) But,

2 The ∠ *a* = ∠ *e*, alſo the ∠ *b* = the ∠ *d*. (*per cor.* 2 *theo.* 29) that is, the ∠ BAD + ∠ *e* + ∠ *d* = 2 ∟s. (*per ax.* 1.) But,

3 The ∠ *e* + ∠ *d* = the ∠ BCD. (*per ax.* 9.) ∴

4 The ∠ BAD + the ∠ BCD = 2 ∟s. *Q. E. D.*

From the ſame manner of argument may the ∠ ABC + the ∠ ADC be proved = 2 ∟s.

THEO. 31. 32 EUC. LIB. 3.

If a right line, as AB, *touch a circle, and another right line, or chord, as* CD, *be drawn from the point of contact* C, *the* ∠BCD *between the chord and tangent, is = to the* ∠ CED *made in the alternate or oppoſite ſegment.* fig. 31.

HYPOTHESIS. THESIS.

AB *is a tangent of the circle* CED *and* CD *is a chord drawn, from the point of contact* C. ∠ BCD *made by the chord and tangent lines will = the* ∠ CED *in the alternate or oppoſite ſegment.*

In

In this theo. are two caſes diſtinguiſhed from the poſition of the chord line.

CASE I. When the chord line is at ∟s to the tangent, conſeq a diameter.

In this caſe the chord line will be ⊥ to the tangent, and the alternate or oppoſite ſegment will therefore be a ſemi-circle, (*per cor.* 2, *theo.* 28) but the ∠ CDE in the ſemi-circle is a ∟, (*per cor.* 3, *theo.* 29) conſeq muſt be = to the ∠ BCD, which by the hypotheſis is a ∟. ∴ &c. *Q. E. D.*

CASE II. When the chord line CD falls obliquely, and divides the circle into two unequal ſegments.

In this caſe, draw the right line CF ⊥ to the tangent AB (146 *caſe* I) this will divide the circle into two = ſegments, (*per cor.* 2, *theo.* 28) and draw DF, then the ∠ CDF in the ſemi-circle will be a ∟ (*per cor.* 3, *theo* 29) and = the ∠ BCF, which is alſo a ∟ by conſtruction; but, the ∠ CDF = ∠ *a* + ∠ *b*, (*per cor.* 5, *theo.* 9) conſeq the ∠ BCF is = ∠ *a* + ∠ *b*; here the ∠ *a* is common, wherefore, taking it away, the remaining ∠ *b* will be = to the ∠ BCD, but the ∠ *b* = the ∠ CED, (*per cor.* 2, *theo.* 29.) ∴ ∠ CED = ∠ BCD. (*per ax.* 1.) *Q. E. D.*

If it were required to prove the obtuſe ∠ ACD = *the obtuſe* ∠ CGD *in the leſſer ſegment, it may be done thus.*

The ∠ ACD + ∠ BCD = 2 ∟s, (*per theo.* 6.) alſo in the quadrilateral CGDE, the oppoſite ∠s G and E together = 2 ∟s, (*per theo.* 30) conſeq ∠ ACD + ∠ BCD = the ∠ G + the ∠ E; (*per ax.* 1) but the ∠ E was proved in the ſecond caſe to be = the ∠ BCD, ∴ the remaining ∠ G muſt be = to the ∠ ACD. *Q. E. D.*

THEO.

THEO. 32. 35 EUC. LIB. 3.

If any two right lines or chords as, AB *and* CD, *be drawn in a circle cutting each other* e. g. *in the point* E, *the rgles* CED *and* AEB, *under their ſegments, will be* =. fig. 32.

HYPOTHESIS. *AB and CD are two chords in a circle cutting each other in* E.

THESIS. *The rgle* CED = *the rgle* AEB.

DEMONSTRATION.

In this theo. are four caſes, diſtinguiſhed by the interſection of the chords.

CASE I. When the chords interſect each other in the center.

In this caſe the theo. muſt be manifeſt, becauſe the rgles would be under the = radii.

CASE II. When one chord, as CD, paſſes through the center, and biſects the other AB not paſſing through the center. fig. 32, No. 1.

Here, becauſe CD biſects AB, it will cut it ⊥ly (*per cor.* 1, *theo.* 27) now, let AF and FB be drawn, then, becauſe CD is biſected in F and otherwiſe divided in E, it will be

1 The rgle $CED + EF^2 = FD^2$ (*per theo.* 21) *i. e.* to FB^2 (for FD = FB). And,

2 $FB^2 = EF^2 + EB^2$ (*per theo.* 16). That is,

3 The rgle $CED + EF^2 = EF^2 + EB^2$, (*per ax.* 1) here EF^2 is common. Conſeq taking it away,

4 The rgle $CED = EB^2$ (*per ax* 3.) *i. e.* to the rgle AEB; (AE being = EB). ∴

5 The rgle CED = the rgle AEB. *Q. E. D.*

CASE

CASE III. When one chord, as CD, paſſes through the center and does not biſect the other AB, but cuts it unequally.

fig. 32, No. 2.

In this caſe, from the center draw the right line FB, and alſo the right line FG ⊥ to AB, (146 *Caſe* 1) which line FG will biſect AB in G, and make the ∠ FGB a ∟, (*per theo.* 27.) Then

1 The rgle CED + EF^2 = FD^2 (*per theo.* 21.) *i. e.* to FB^2, (for FD = FB). But,

2 FB^2 = FG^2 + GB^2 (*per theo.* 16) and for the ſame reaſon will

3 EF^2 = FG^2 + GE^2. That is,

4 The rgle CED + FG^2 + GE^2 = FG^2 + GB^2, (*per ax.* 1) here FG^2 is common, and by taking it away from 4th ſtep, there will remain

5 The rgle CED + GE^2 = GB^2. (*per ax.* 3.) But,

6 GB^2 = the rgle AEB + GE^2, (*per theo.* 21) *i. e.* the rgle CED + GE^2 = the rgle AEB + GE^2. (*per ax.* 1.) ∴ by taking away the common □ GE, there will remain

7 The rgle CED = the rgle AEB. *Q. E. D.*

CASE IV. When neither CD nor AB paſs through the center. fig. 32, No. 3.

To prove this, draw the diameter GF *through the point of interſection* E. Then,

1 The rgle GEF = the rgle CED. (*per caſe* 3 *of this.*) Alſo,

2 The rgle GEF = the rgle AEB (*per the ſame.*) ∴

3 The rgle CED = the rgle AEB. (*per ax* 1.)

Q. E. D.

For another more elegant demonſtration of this theo. ſee part the firſt, theo. 46.

THEO.

THEO. 33. 36 EUC. LIB. 3.

If from any point, as A, *without a circle, there be drawn two right lines, one of them, as* AB, *cutting it in* E, *and the other,* AD, *touching it in* D, *the rgle* BAE *under the whole ſecant line* AB, *and that part of it,* AE, *without the circle, will* = *the* □ *of the tangent* AD. fig. 33.

HYPOTHESIS.

A is a point without a circle, from whence the right line AB *is drawn, cutting it in* E, *and* AD *is another right line touching it in* D.

THESIS.

The rgle BAE = *the* □ *of* AD.

DEMONSTATION.

In this theo. there are two caſes, ariſing from the poſition of the ſecant line.

CASE I. When the ſecant line is drawn through the center.

In this caſe, let the right line CD be drawn from the center E, to the point of contact D, this will be ⊥ to the tangent AD, (*per part* 1, *theo.* 28) and, becauſe BE is biſected in C, and to it AE is added. Then,

1 The rgle $BAE + CE^2 = CA^2$ (*per theo.* 22) *i. e.*

2 The rgle $BAE + CD^2 = CA^2$, (becauſe CD = CE). But,

3 $CA^2 = CD^2 + AD^2$, (*per theo.* 16.) Conſeq

4 The rgle $BAE + CD^2 = CD^2 + AD^2$ (*per ax.* 1.) ∴ by taking away the common □, CD, there will remain

5 The rgle $BAE = AD^2$. *Q. E. D.*

CASE

CASE II. When the ſecant line doth not paſs through the center.

In this caſe, through the centre C, draw the line AC and alſo the radii CD and CE, and laſtly CF biſecting BE in F, conſeq. the ∠CFE will be a ∟. (*per cor.* 1, *to theo.* 27.) Then,

1 The rgle $BAE + EF^2 = FA^2$, (*per theo.* 22.) to which add FC^2. Then (*per ax.* 2.)

2 The rgle $BAE + EF + FC^2 = FA^2 + FC^2$. (*i. e. to* AC, *per theo.* 16.) But,

3 $EF + FC^2 = EC^2$ or CD^2, (*per theo.* 16.) for EC = CD. Conſeq.

4 The rgle $BAE + CD^2 = AC^2$, (*i. e.* to $CD^2 + AD^2$.) (*per theo.* 16.) ∴ by taking away CD^2 it being common.

5 The rgle $BAE = AD^2$. (*per ax.* 3.) *Q. E. D.*

COROLLARY 1.

Hence, if from any point without a circle there be drawn two right lines cutting the circle and terminating in the concave periphery, the rgle under the whole ſecant and part without the circle, will = the rgle under the other ſecant line and part without. *For if a tangent line be drawn from the point, each rgle will be = to the □ of the tangent;* ∴ *the rgles muſt be =.* (per ax. 1.)

The contrary of this cor. muſt be true; *i. e.* if two tangents be drawn to any circle from a point, without the circle, their □s will be =, and the tangents alſo =, *for if a ſecant line be drawn from the ſame point the □ of each tangent will be = to the rgle under the whole ſecant line and part without the circle,* ∴ *&c.*

THEO. 34. 15 EUC. LIB. 5.

Quantities and their like multiples, (107) *have the ſame ratio.*

HYPOTHESIS. THESIS.

The quantities 2 A 2 B *are equimultiples of* A *and* B. A : B :: 2A : 2B :: 3A : 3B, &c.

DEMONSTRATION.

If it be denied that the ratio of A : B = 2A : 2B, &c. *i. e.* = to the ratios of like multiples of A and B, then the ratio of A : B muſt = the ratio of CA : DB, or ſome unlike multiples of them. (*The quantities* C *and* B *being any quantities at pleaſure.*) Now becauſe it is (*by ſuppoſition*) A : B :: CA : DB, then by alternate ratio (115) it will be A : CA :: B : DB. *i. e.* $\frac{A}{CA} = \frac{B}{DB}$ (*per def.* 113) wherein the conſequents CA and DB are not equimultiples of their antecedents A and B; but the ratio of $\frac{A}{CA}$ is not = to the ratio of $\frac{B}{DB}$, becauſe the quantities C and D are unequal; conſeq the proportion A : CA :: B : DB is not true, neither is A : B :: CA : DB true, ∴ the ratio of the quantities A and B, and their like multiples 2 A and 2 B are =, that is, A : B :: 2 A : 2 B :: 3 A : 3 B, &c. (*See note to def.* 132.) *Q. E. D.*

COROLLARY 1.

Hence the firſt of four proportional quantities is ſaid to have the ſame ratio to the ſecond, which the third has to the fourth, when any equimultiples of the firſt and third being taken, as alſo, any equimultiples of the ſecond and fourth, for *if the multiple of the firſt*, be leſs than, equal to, or greater *than the ſecond, in like manner will the multiple of the third* be leſs than, equal to, or greater than the fourth, *as in the quantities,* 2 6 8 24 A : B :: C : D where 4 12 24 72 2A : 2B :: 3C : 3D, *and ſo of others.* (5 def. Euc. lib. 5.)

COR. 2. Hence in any ratio, if the ſame quantity or quantities be contained in both terms, the value of the

the ratio, when theſe quantities are taken away, will be ſtill the ſame, *for* $\frac{a\,c}{b\,c} = \frac{a}{b}$ *the quantity* c *being taken away.*

Cor. 3. Quantities and their like parts have equal ratios, *for* a *and* b *are like parts of* C a *and* C b, (*i. e.* C × a and C × b,) ∴ C a : C b = a : b.

Cor. 4. Quantities and their like multiples, or like parts are proportional, *for* A : B :: c A : c B, ∴ *by inverſion* c A : c B :: A : B. (116.)

Cor. 5. If quantities be =, their like multiples or like parts will be alſo =, *for if* A = B, *then will* $\frac{A}{B} = \frac{c\,A}{c\,B}$, *that is, the antecedents and conſequents are in a ratio of equality.* (111.)

Cor. 6. If the parts of any quantity be proportional to the parts of any other quantity, they will alſo be like parts of their reſpective quantities, and the contrary, *for only like parts are proportional to their wholes.* (Per cor. 3, to this.)

Cor. 7. What ratios are = to the ſame ratio are = to each other, *for* $\left\{\begin{matrix} \textit{if } A : B = CA : CB \\ \textit{and } A : B = DA : DB \end{matrix}\right\}$ *then will* CA : CB = DA : DB. (11 E. Lib. 5.)

Cor. 8. Proportions which are the ſame, to the ſame proportion are the ſame to each other; for if $\left\{\begin{matrix} A : B :: C : D \\ \text{and } A : B :: E : F \end{matrix}\right\}$ then will C : D :: E : F. (*Per cor.* 7.)

Cor. 9. If two ratios or products be =, their like multiples by the ſame or = quantities, or by = ratios, will be = alſo; *i. e.* if A : B = C : D, *then will* AE : BE = CE : DE, *now ſuppoſe the quantity* E = *any other quantity, as* F, *then will* AE : BE = CF : DF; *and laſtly, if* E : F :: G : H, *then will* AE : BF = CG : DH, *for in all theſe examples the ratios may be taken or conſidered as quantities.*

Theo.

THEO. 35. 7 EUC. LIB. 5.

Equal quantities A *and* B, *have the ſame ratio or proportion to another quantity* C, *and the contrary;* i. e. *any quantity, as* C, *will have the ſame ratio to* = *quantities.* fig. 35.

HYPOTHESIS.

A *and* B *are two* = *quantities, and* C *is a third quantity.*

THESIS.

A : C :: B : C, *and* C : A :: C : B.

DEMONSTRATION.

Becauſe A = B, then each muſt have the ſame ratio to C, (*per cor.* 4, *to theo.* 34) *i. e.* C will be the ſame part or multiple of A, that it is of B.

for A : B :: C : C. (*per cor.* 4, *theo.* 34)
and C : C :: A : B. (*per ſame.*

∴ A : C :: B : C. (*per def.* 115.
alſo C : A :: C : B. (*per ſame.* Q. E. D.

COROLLARY 1.

When the antecedents of any ratio are =, the conſequents will be = alſo.

Cor. 2. When quantities have the ſame ratio, to, or are like multiples or like parts of any other quantities, they will be =, for, if A : C :: B : C, *then will the quantities* A *and* B *be* =.

*** On this theo. doth the inverſion of ratio depend, for, ſince A : C :: B : C and C : A :: C : B, (*per this theo.*) then, is this latter proportion ſaid to be the inverſion of the former. (*def.* 116.)

THEO. 36.

In two or more ſets of proportional quantities, the rgle under like terms are proportional. fig. 36.

HYPO-

HYPOTHESIS. THESIS.

A : B :: C : D, *and* E : F :: G : H. AE : BF :: CG : DH.

DEMONSTRATION.

Since A : B : : C : D } (*by hypo.*) then will AE : BF
and E : F : : G : H }
: : CG : DH. (*per cor.* 9, *theo.* 34.)

THEO. 37. 16 EUC. LIB. 6.

In four proportional quantities . A . C . D . *and* B, *the rgle under the two extremes,* (i. e. *the first and last*) *will* = *the rgle under the two means,* (i. e. *the middle terms.*) fig. 37.

HYPOTHESIS. THESIS.

In the ▭*s* X *and* Z, A : C :: D : B, i. e. *length is to length as breadth is to breadth.* *The rgle* AB *under the extremes* = *the rgle* CD *under the means.*

DEMONSTATION.

Since the ratio of A : C = D : B, by hypo. then will the ratio A : C = AB : CB, (*per cor.* 4, *theo.* 34) and for the same reason the ratio of D : B = CD : CB, conseq the ratio of AB : CB = CD : CB, (*per cor.* 8, *theo.* 34) but, here the consequents CB and CB are the same, ∴ taking them away the antecedents AB and CD will remain =, *i. e.* the rgle AB = the rgle CD. *Q. E. D.*

*** *Note, this theo. was demonstrated before in* rem. to def. 117, *but it was thought necessary to give it a more geometrical demonstration here.*

COROLLARY 1.

Hence if the rgle or product of any two quantities be = to the rgle or product of any other two quantities, then

then will the four quantities be proportional; for ſuppoſe the □s X and Z be =, as was juſt proved, *i. e.* $\overline{A \times B} = \overline{C \times D}$, then will A and C their lengths, and B and D their breadths be proportional; or A : C :: D : B; and ſuch figures are called reciprocal figures, in which is to be obſerved, their lengths are reciprocally to each other as their breadths, or, the antecedents and conſequents of the two ratios are in both figures, *i. e.* the length of X is to the length at Z reciprocally, as the breadth of X to the breadth of Z, *i. e.* A : C :: D : B and the proportion of A : C :: D : B is reciprocal of the proportion A : B :: C : D.

Cor. 2. Hence, if any two △s or □s, &c. have their baſes and altitudes reciprocal y proportional, they will be = and the contrary, that is, if the figures be =, their baſes and altitudes will be reciprocally proportional *.

Cor. 3. Hence alſo, any □s, as X and Z, having an ∠ *a* in one = to an ∠ *b*, in the other the ſides about the = ∠s will be reciprocally proportional, *i. e.* A : C :: D : B; the ſame muſt be true of △s, *for by drawing diagonals in the □s oppoſite to the ∠s, then will the ſides about the = ∠s be reciprocally proportional, as well in the ∠s, as in the □s, ∴ &c.*

Theo. 38. 16 Euc. Lib. 6.

Any △s whatſoever, as BDE *and* EDC, *and alſo □s, or* BK *and* EL *having the ſame altitude, or being between the ſame or equidiſtant ‖s, are in proportion to each other as their baſes.* fig. 38.

* Reciprocal proportion being accidentally omitted among the definitions, it may be here explained, *thus, among four proportional quantities, when the 4th term is leſs than the ſecond, by ſo much as the third is greater than the 1ſt, the quantities are ſaid to be reciprocally proportional,* as 4 : 10 :: 8 : 5.

Hypo-

Hypothesis.	Thesis.
BDE *and* EDC *are* Δ*s, alſo* BK *and* EL *are* □*s having the ſame altitude, or between the ſame* ‖*s* GI *and* AL.	1 ΔBDE : ΔEDC :: BE : EC. 2 □BK : □EL :: BE : EC.

To prove this, let the line BC be produced both ways to G and I, ſo that BF and FG ſhall each = BE, and alſo that CH and HI ſhall each = EC, and draw the lines FG, GD, HD and ID as in the figure.

Demonstration.

The Δs GDF, FDB and BDE are =, (*per cor. to theo.* 15) and for the ſame reaſon the Δs EDC, CDH and HDI are alſo =, conſeq the Δ GDE and its baſe GE are like or equimultiples of the Δ BDE and its baſe BE, (*per def.* 107) for the Δ GDE is treble the Δ BDE, and alſo the baſe GE, is treble the baſe GF, and in like manner the Δ EDI and its baſe EI are like or equimultiples of the Δ EDC and its baſe EC, wherefore it will be

as Δ GDE : Δ BDE :: GE : BE
alſo Δ EDI : Δ EDC :: EI : EC } *per def.* 111.

For whatever proportion is found to ſubſiſt between the wholes, the ſame muſt alſo ſubſiſt between their like parts. (*per cor.* 4, *theo.* 34.)

∴ the Δ BDE : Δ EDC :: BE : EC, (*per cor.* 1, *theo.* 34.) *Q. E. D. in the firſt part.*

Again, as to the □s, becauſe the Δs BDC and EDC are reſpectively halves of the □s BK and EL, it muſt follow, that

Δ BDE : Δ EDC :: □ BK : □ EL. (*per theo.* 34.) But,

Δ BDE : Δ EDC :: BE : EC ∴ □ BK : □ EL :: BE : EC, (*per cor.* 7, *to theo.* 34.) *Q. E. D. in the 2d part.*

Corollary.

Hence, all □s and Δs having the ſame or = baſes, are in proportion to each other as their altitudes, for,

by taking their = baſes as their altitudes; or *conſtructing other □s and △s equal to them, the altitude of which ſhall be their equal baſes, in either caſe the cor. will be manifeſt.*

THEO. 39. 2 EUC. LIB. 6.

In any plain △, *as* ABC, *if any two adjoining ſides, as* AB *and* AC, *be cut by a line, as* ED, *drawn* ‖ *to the remaining ſide* CB, *the ſegments of theſe lines will be proportional.*

fig. 39.

HYPOTHESIS.	THESIS.
ABC *is a* △, *the ſides of which, viz.* AB *and* AC *are cut by the line* DE *drawn* ‖ *to* CB.	AD : DB :: AE : EC.

To prove this draw the lines EB and DC.

DEMONSTRATION.

1 The △ AED : △ DEB :: AD : DB, (*per theo.* 38.) Again,

2 The △ AED : △ EDC :: AE : EC, (*per ſame*) But,

3 △ DEB = △ EDC, (*per cor. to theo.* 15.) ∴

4 AD : DB :: AE : EC. (*per cor.* 1, *theo.* 34.) *Q. E. D.*

COROLLARY 1.

Hence, if any line, as DE, be drawn in any △ ſo as to cut any two ſides proportionally, that line will be ‖ to the remaining ſide.

1 For AD : DB :: △ ADE : △ DEB. (*per theo.* 38.) Again,

2 AE : EC :: △ ADE : △ EDC. (*per the ſame.*) But,

3 AD : DB :: AE : EC. (*per this theo.*) Conſeq

4 △ ADE : △ DEB :: △ ABE : △ EDC. (*per cor.* 7, *theo.* 34.) Wherefore,

5 △ DEB

5 Δ DEB = Δ EDC, *(per cor.* 1, *theo.* 35) but theſe Δs have a common baſe, viz. ED, ∴ the lines ED and CB muſt be ||, *(as may appear from the cor. to theo.* 15.)

Cor. 2. When the ſides of any Δ, as ABC, are cut proportionally, the ſegments of thoſe ſides, viz. AD, AE, DB and EC will be proportional to the ſides; *i. e.* AD : AB :: AE : EC, alſo BD : BA :: CE : CA.

1 For Δ AED : Δ AEB :: AD : AB. *(per theo.* 38.) Alſo,

2 Δ AED : Δ ADC :: AE : AC. *(per ſame.)* ∴

3 AD : AB :: AE : AC. *(per cor.* 8, *to theo.* 34) and in the ſame manner it may be ſhewn that BD : BA :: CE : CA.

Theo. 40. 1 Part of 3 Euc. Lib. 6.

If any right line, as BF, *which biſects an* ∠ *of any* Δ, *as* ABE, *and ſhall alſo cut the baſe* AE *in* F, *then will the ſegments* AF *and* FE *of the baſe have the ſame proportion to each other, that the lines* AB *and* BE *have, which are about the biſected* ∠. fig. 40.

Hypothesis.	Thesis.
BF *is a right line which biſects the* ∠ B *of the* Δ ABE, *and cut the baſe* AE *in* F.	AF : FE :: AB : BE.

To prove this, draw forth AB unto D, ſo that BD ſhall = BE and draw DE.

Demonstration.

Then, becauſe the Δ DBE is an iſoceles by conſtruction, the ∠ *e* = ∠ D, *(per theo.* 5) again, the external ∠ ABE is equal to the two internal ∠s *e* and D, *(per theo.* 9) *i. e.* the ∠ *b* which is half ABE *(by hypo.)* ſhall = the ∠ D, and the ∠ *a* = ∠ *e* for the ſame reaſon; wherefore, BF and DE are ||, *(per cor.* 1, *to theo.* 8) wherefore, in the Δ ADE, becauſe BF

and DE are ||, AF : FE :: AB : BD; *(per theo.* 39) but BD = BE by conſtruction, ∴ AF : FE :: AB : BE. *Q. E. D.*

COROLLARY.

Hence, in any △, if a right line be drawn from any of its ∠s, and cut the oppoſite ſide, ſo as to make the ſegments of that ſide have the ſame proportion between themſelves that the ſides about the ∠ have, this right line will biſect the ∠. *Second part of* 3 *E. lib.* 6.

Which, if it be denied, it may be proved from figure 40, *conſtructed as before; thus, becauſe* AF :: FE :: AB : BD, (i. e. BE *per conſtruction*) B F *and* D E *muſt be* || *(per cor.* 1, *theo.* 39) conſeq the external ∠ *b* = the internal ∠ D, and the ∠ *a* = ∠ *e* being alternate; *(per theo.* 8) but, the ∠ D = ∠ *e*, the △ BDE being an iſoceles, ∴ ∠ *b* = ∠ *a*, conſeq the ∠ ABE is biſected.

THEO. 41. 4 EUC. LIB. 6

Any triangles as ABC *and* abc, *which are equiangular or ſimilar to each other, will have their ſides, which are about* = ∠*s, and alſo, the ſides which are oppoſite to the* = ∠*s proportional.* fig. 41.

HYPOTHESIS. THESIS.

The △*s* ABC *and abc are ſimilar.*

1 { *ba* : *bc* :: BA BC } *i. e. the ſides*
{ *ba* : *ac* :: BA AC } *about* = ∠*s are*
{ *ac* : *cb* :: AC CB } *proportional.*

2 *ab* : AB :: *bc* : BC, *i. e. the ſides oppoſite* = ∠*s are proportional.*

To prove this, take in the ſides BA and BC of the △ ABC, the interval BE = *bc*, *alſo* BD = *ba*, and draw DE.

DEMONSTRATION.

1 Becauſe the △s BDE and *bac* are congruous, *(per theo.* 1) the ∠ BDE = ∠ *a*, *(i. e. by hypo.)* to the ∠ A, conſeq DE is || to AC, *(per cor.* 1, *theo.* 8) wherefore,

2 BD

2 BD : BA :: BE : BC, (*per cor.* 2, *theo.* 39.) But,

3 BD = *ba* & BE = *bc*, (*per conſtruction.*) Conſeq

4 *ba* : BA :: *bc* : BC, That is,

5 *ba* : *bc* :: BA : BC. (*per def.* 115.)

In the ſame manner you may prove that *ba* : *ac* :: BA : AC, and alſo, *ac* : *cb* :: AC : CB. *Q. E. D. in the firſt part.*

The ſecond part of this theo. muſt be true from the matter produced in the 4th ſtep of the 1ſt part, where it was proved that *ab* ; AB :: *bc* : BC ; ∴ &c. *Q. E. D. in the ſecond part.*

COROLLARY I.

Hence △s having one ∠ in each =, and the ſides about thoſe = ∠s proportional, theſe △s will be equiangular and ſimilar. 6 Euc. lib. 6.

COR. 2. Hence alſo, all ſimilar figures, whether polygons or △s, will always have the ſides about their = ∠s proportional, and the contrary; *i. e.* if the ſides about = ∠s be proportional, the figures will be ſimilar.

THEO. 42. 8 EUC. LIB. 6.

In any right ∠'d △, *as* ABC, *if a* ⊥, *as* BD, *be drawn from the* ∟ B *to the oppoſite ſide* AC, *it will divide the* △ *into two* △s ADB *and* BDC, *ſimilar to each other, and to the whole* △ ABC. fig. 42.

HYPOTHESIS.

The △ ABC *is a* ∟*'d* △, ∠*'d at* B, *and* BD *is a right line, let fall* ⊥*ly from the* ∟ B *to the oppoſite ſide* AC.

THESIS.

The △s ABC, ADB, *and* BDC *are ſimilar to each other.*

DEMONSTRATION.

In the △s ABC and ADB the ∠ A is common to both; again, the ∠ ABC = the ∠ ADB being both ∟s;

∟s; wherefore, the remaining ∠ C will = ∠ ABD, (*per cor.* 3, *theo.* 9) conſeq the △ ABC is ſimilar to the △ ADB; (*per def.* 75) in the ſame manner may the △ BDC be proved ſimilar to the △ ABC; therefore, the △s ABC, ADB, and BDC are ſimilar. *Q. E. D.*

COROLLARY 1.

The ⊥ BD is a mean proportional *, between the ſegments AD and DC, for AD : DB :: DB : DC. (*per theo.* 41.)

COR. 2. The ſide AB is a mean proportional between the ſide AC and the ſegment AD, for AC : AB :: AB : AD. (*per theo.* 41.)

COR. 3. The ſide BC is a mean proportional between the ſide AC and the ſegment DC, for AC : BC :: BC : DC. (*per theo.* 41.)

COR. 4. Hence alſo, if a right line be let fall from any point of the circumference of a circle ⊥ to the diameter, it will be a mean proportional between the ſegments of the diameter; for, *let lines be drawn from the point from which the ⊥ falls to the extremities of the diameter, and a ∟'d △ will be formed,* (per cor. 3, theo. 29) *conſeq by* (cor. 1, of this theo.) *the ⊥ will be, &c.*

THEO. 43. 19 EUC. LIB. 6.

Similar △s, *as* ABC *and* DEF, *are to each other in a duplicate ratio* (119) *of the homologous ſides; i. e. the ſides which are oppoſite to, or ſubtend* = ∠s. fig. 43.

* *A mean proportional* between two quantities, is that quantity, the ſquare of which will = the product of the two quantities; thus, 8 is a mean proportional between 4 and 16, for 4 : 8 : 8 :: 16, *i. e.* the ſquare of 8 (viz. 64) = 4 × 16, &c.

HYPO-

HYPOTHESIS.

The Δs ABC *and* DEF *are ſimilar; and* CB *and* EF, *alſo,* AB *and* DE *are homologous ſides.*

THESIS.

Δ DEF : Δ ABC :: $FE^2 : CB^2$, *that is, in a duplicate ratio of the ſide* EF *and* CB.

To prove this in the line EF, find the point G *(per prob.* 47) ſo that GF may be a third proportional between EF and BC, and draw the line DG, then it will be FE : CB :: CB : GF.

DEMONSTRATION.

1 Since the Δs DEF and ABC are ſimilar, *(by hypo.)* DF : FE :: AC : CB, *(per theo.* 41.) *i. e.*

2 DF : AC :: FE : CB, *(per def.* 115) but by conſtruction, FE : CB :: CB : FG. Conſeq it will follow,

3 DF : AC :: CB : FG, *(per cor.* 7, *theo.* 34) *i. e.* in the Δs ABC and DGF, the ſides about the = $\angle$s C and F are reciprocally proportional, conſeq the Δs are =. (*per cor.* 3, *theo.* 37.) But,

4 The Δ DEF : Δ DGF : EF : GF, (*per theo.* 38) and the Δ DGF = Δ ABC, *(per laſt ſtep.)* Conſeq

5 The Δ DEF : Δ ABC :: EF : GF, but by conſtruction EF, CB and FG are three proportional terms, wherein, EF and FG are are in a duplicate ratio of EF and CB; *(per def.* 119) that is, EF : FG :: $EF^2 : CB^2$. ∴

6 The Δ DEF : Δ ABC : $EF^2 : CB^2$. *Q. E. D.*

COROLLARY.

Hence, if any three lines be proportional, it will be as the firſt is to the third, ſo is any figure conſtructed upon the firſt, to a ſimilar figure conſtructed upon the ſecond, for the lines EF, CB, *and* FG *are proportional, conſeq* EF : FG :: *any* Δ, $\square$, *&c. made upon* EF *to a ſimilar* Δ, $\square$, *&c. made upon* CB, i. e. *(per def.* 119) EF : FG :: $EF^2 : CB^2$.

THEO.

THEO. 44. 20 EUC. LIB. 6.

Like polygons ABCDE, *and* FGHIK, *are divided by the diagonals into an* =, *No. of ſimilar* Δ*s, viz.* a, d; b, e; *and* c, f; *in the ſame proportion to each other as the polygons; and polygon is to polygon in a duplicate ratio of their homologous ſides.* fig. 44.

HYPOTHESIS.

The polygons ABCDE *and* FGHIK *are ſimilar, and* EB, EC, *alſo,* KG *and* KH *are diagonal lines in each, which lines divide each polygon into* Δ*s.*

THESIS.

1 *The polygons are divided into an* = *number of* Δ*s ſimilar to each other.*

2 *Theſe* Δ*s have to each other the ſame ratio, that polygon has to polygon.*

3 *Polygon is to polygon in a duplicate ratio of their homologous ſides, viz.* AB *and* FG, *i. e. as* $AB^2 : FG^2$.

DEMONSTRATION.

1 Becauſe the polygons are ſimilar they will be equiangular, *(per def.* 75) viz. ∠ A = ∠ F; ∠ B = ∠ G, &c. and the ſides about the = ∠s will be proportional; *(per theo.* 41) *i. e.* BA : AE :: GF : FK, ∴ the Δs *a* and *d* are ſimilar, *(per cor.* 1, *theo.* 41) in the ſame manner the Δs *e* and *f* may be proved ſimilar; again, the ∠ CBA = ∠ HGF *(per def.* 75) from both take the = ∠s ABE and FGK, and the remaining ∠s EBC and KGH will be = alſo, in like manner the ∠s ECB and KHG may be proved =, and the ∠ CEB = ∠ HKG *(per cor.* 3, *theo.* 9) conſeq the Δs *b* and *c* are ſimilar; ∴ the polygons are divided into an = number of ſimilar Δs. 2. *E. D. in the firſt part.*

2 Becauſe the Δs *a* and *d* are ſimilar, the Δ *a* : Δ *d* : $EB^2 : KG^2$, *(per theo.* 43) for the ſame reaſon the Δ *b* : Δ *e* :: $EB^2 : KG^2$, conſeq Δ *a* : Δ *d* :: Δ *b* : Δ *e*,

: $\triangle e$, (*per cor.* 7, *to theo.* 34) in the ſame manner it may be ſhewn that the $\triangle c : \triangle f :: \triangle b : \triangle e$; but, as the whole polygons are = to the ſum of their reſpective $\triangle$s, ∴ as any one $\triangle$ in one polygon, (viz. the $\triangle a$) is to it a ſimilar $\triangle$ in the other polygon, (viz. the $\triangle d$) ſo is the ſum of all the $\triangle$s in one polygon to the ſum of all the $\triangle$s in the other; *i. e.* as $\triangle : \triangle ::$ polygon to polygon. *Q. E. D. in the ſecond part.*

3 Becauſe $\triangle a : \triangle d :: AB^2 : GF^2$; (*per theo.* 43) *i. e.* in a duplicate ratio of their like ſides, and as the proportion of polygon to polygon is, as $\triangle a : \triangle d$ as was ſhewn in the ſecond part, ∴ polygon : polygon $:: AB^2 : GF^2$, *i. e.* in a duplicate ratio of their like ſides. *Q. E. D. in the third part.*

Corollary 1.

Hence, all regular polygons or ſimilar figures whatever, as equilateral $\triangle$s, $\square$s, &c. are to each other in a duplicate ratio of their like ſides.

*** From this cor. the 16th theo. or 47 Euclid lib. 1, is univerſal, for not only $\square$s but whatever ſimilar figures are made upon the ſides of any $\llcorner$'d $\triangle$, the area of that on the ſide oppoſite the $\llcorner$ will be = to the ſum of the areas of thoſe ſimilar, figures made on the legs about the $\llcorner$; *for, the figures will be to each other in a duplicate ratio of their like ſides;* i. e. *as the* $\square$s *of the ſides of the* $\triangle$*s, &c.*

Cor. 2. Hence alſo, if the ſides of any ſimilar figures which are between the $=\angle$s be known, the proportion of the figures is alſo known; *thus, if the ſide of one figure be* 2, *and that of the other be* 6, *then it will be* : 2 : 6 :: 6 *to a third proportional, viz.* 18, *conſeq the proportion of figure to figure will be as* 2 : 18 *or as* 1 *to* 9.

Cor. 3. Hence, may all rectilineal figures whatſoever be increaſed or diminſhed in a given proportion; *for, take a line as much greater, or leſs, than one of the ſides of the figure to be increaſed, &c. as the given proportion, and between*

tween that line, and the ſide of the given figure find a mean proportional (per prob. 49) *on which mean proportional, conſtruct a ſimilar figure* (per prob. 46) *and that will be the figure which was required.* (See prob. 56.)

THEO. 45. 47 EUC. LIB. 1*.

In all ∟ △*s, as* ABC, *the* □ *of the hypothenuſe or ſide oppoſite the* ∟ *is* = *to the ſum of the* □*s of the other two ſides.* fig. 45.

HYPOTHESIS. THESIS.

The figure ABC *is a* ∟*d* △, ∟*'d at* B. $AC^2 = AB^2 + BC^2$.

To prove this conſtruct □s on the three ſides (*per prob.* 12) and draw the line BD ⊥ to AC, which produce to F. Then

DEMONSTRATION.

1 AC : AB :: AB : AD, (*per cor.* 3, *theo.* 42) Conſeq

2 $AC \times AD = AB^2$, *i. e.* the □ AF = AB^2, (*per theo.* 37.) Again,

3 AC : BC :: BC : DC (*per cor.* 2, *theo.* 42.) Conſeq

4 $AC \times DC = BC^2$, *i. e.* the □ FC = BC^2. (*per theo.* 37.) But,

5 $AC^2 = \square AF + \square EC$. (*per conſtruction and theo.* 18.) ∴

6 $AC^2 = AB^2 + BC^2$: (*per 2d and 4th ſteps.*)

Q. E. D.

THEO. 46.

If in a circle two chords, as AB *and* CD, *interſect each other in* E *within the circle,* (*by*

* This is the 16th theo. demonſtrated another way agreeable to promiſe in page 47.

pro-

prolonging them) without the circle the rgles under their ſegments terminated by the circumference and their interſection will be =. fig. 46.

HYPOTHESIS.

AB *and* CD *are two chords in a circle cutting each other in* E, *either within or without the circle.*

THESIS.

The rgle AEB = *rgle* CED.

DEMONSTRATION.

Firſt, when the chords interſect within the circle (*as in fig.* 46, *No.* 1) this muſt be true (*per caſe* 4, *theo.* 32) but, for another proof, as promiſed in page 175, let the line BC and AD be drawn as in the figure, then the △s BEC and DEA are ſimilar, for the ∠s at E are =, (*per theo.* 7) and the ∠ *c* = the ∠ A, (*per cor.* 2, *theo.* 29) and for the ſame reaſon the ∠ B = ∠ D, conſeq AE : CE :: DE : BE (*per theo.* 41) ∴ AE × EB = CE × ED, (*per theo.* 37) *i. e.* the rgle AEB = the rgle CED.

Q. E. D.

Secondly, When the chords on being produced interſect each other without the circle, (*as in fig.* 46. *No.* 2) then the △s BEC and DEA are ſimilar, for ∠ is common and ∠ B = ∠ D, (*per cor.* 2, *theo.* 29) conſeq the remaining ∠s A and C muſt be = (*per cor.* 3, *theo.* 9) ∴ (*per theorems* 41 *and* 37, *as in the firſt part,*) the rgle AEB = the rgle CED.

Q. E. D.

THEO. 47.

If in a circle two right lines or chords, AB and CD, interſect each other, as in E, their ∠ of inclination, viz. BEC and AED is meaſured by half the ſum of the intercepted arches, AC and BD. fig. 47.

HYPOTHESIS. THESIS.

AB *and* CD *are two chords in a circle interſecting each other in* E. ∠ BEC *or* AED *is meaſured by half the ſum of the arches* AD *and* BD *which ſubtend them.*

DEMONSTRATION.

Let the line BD be drawn, then will the ∠ CEB = ∠ *a* + ∠ *c*, (*per theo.* 9) but the ∠ *a* is meaſured by half the arch BC (*per cor.* 1, *theo.* 29) and the ∠ *c* is meaſured by half the arch AD (*per the ſame*) ∴ the ∠ BEC (= ∠s *a* and *c*) or its = ∠ AED is meaſured by half the ſum of the arches BC and AD. (*per theo.* 7.) *Q. E. D.*

THEO. 48.

If any right line, as BE, *biſect any* ∠ *of a* △, *as* ABC, *and meets the oppoſite ſide, the* □ *of the ſaid line will* = *the rgle under the ſides about the biſected* ∠, *viz.* AB × BC, — *the rgle under the ſegments of the ſide oppoſite the biſected* ∠, *viz.* AE × EC. fig. 48.

HYPOTHESIS. THESIS.

In the △ ABC *the* ∠ B *is biſected by the line* BE, *which meets the ſide* AC *in* E. BE^2 = *rgle* AB × BC — *rgle* AE × EC.

To prove this, about the △ deſcribe a circle (*per prob.* 20) and continue the line BE to D and join CD, then will the △s ABE and DBC be ſimilar, for the ∠s A and D are =, (*per cor.* 2, *theo.* 29) and the ∠s ABE and DBC are =, by hypo. conſeq the remaining ∠s C and E are =. (*per cor.* 3, *theo.* 9.)

DEMON-

DEMONSTRATION.

1 AB : BE :: DB : BC, (*per theo.* 41.) Conſeq

2 AB × BC = BE × DB, (*per theo.* 37.) But,

3 BE × DB = DE × BE + BE², (*per theo.* 19.) Wherefore,

4 AB × BC = DE × BE × BE², (*per ax.* 1.) But,

5 DE × BE = AE × EC, (*per theo.* 32.) Conſeq

6 AB × BC = AE × EC + BE², (*per ax.* 1.) ∴

7 BE² = AB × BC — AE × EC, (*per ax.* 3) *i. e.* by taking the rgle AE × EC from both ſides of the equation * in the 6th ſtep. *Q. E. D.*

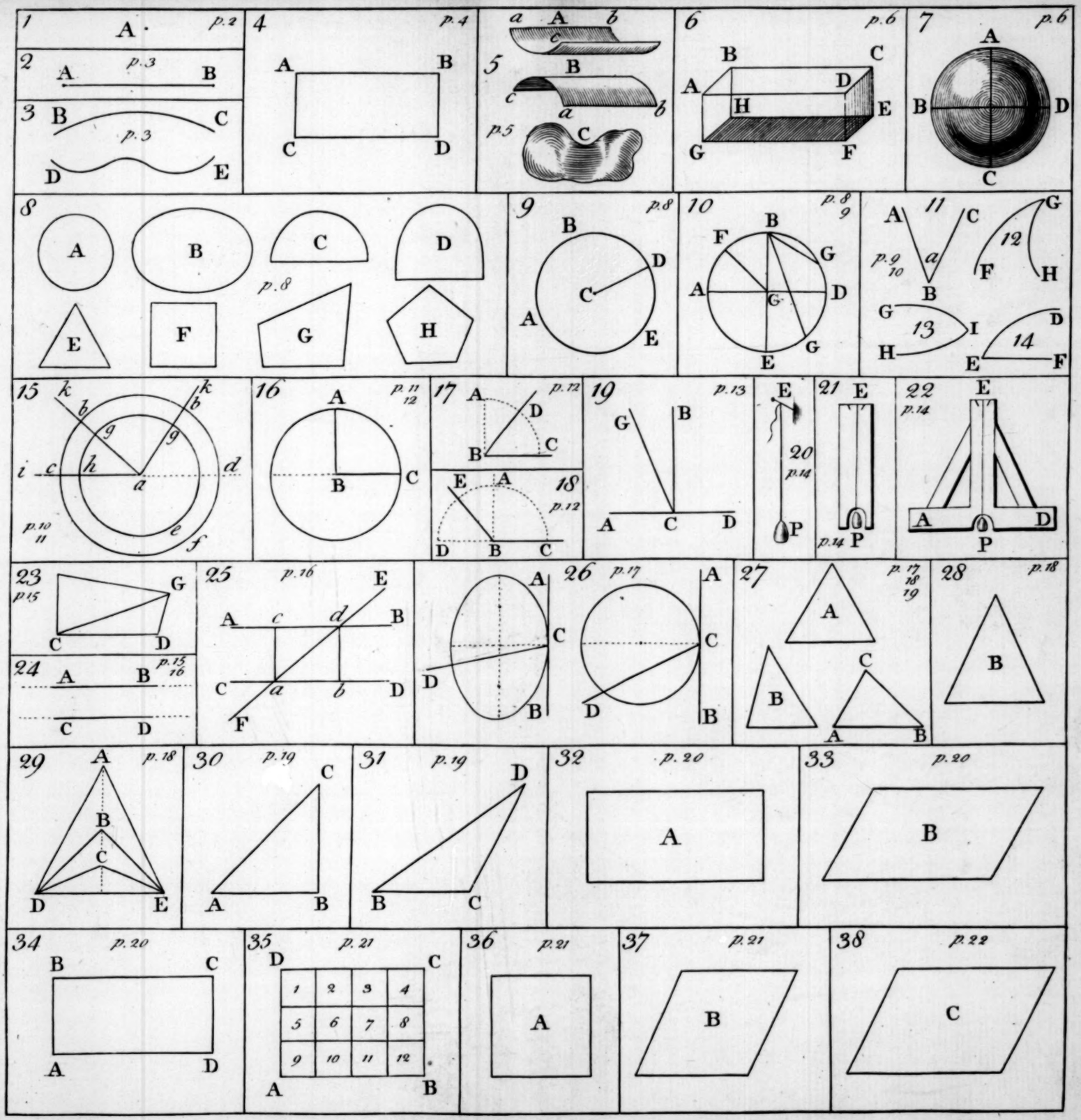
*** *To a reader unacquainted with algebra, the laſt ſtep of this demonſtration muſt appear difficult; i. e. where the rgle* AE × EC *is taken from the 6th ſtep: in order to make this clear to ſuch a reader, he muſt obſerve, that to take any quantity from another repreſented by letters as above, algebraiſts always place the quantity to be ſubtracted, with a negative ſign before it in a new ſtep, as in the above demonſtration* (ſtep 7) *where it is plain, if the rgle* AE × EC, *be taken from* BE² *in the 6th ſtep, the* □ BE *will be left alone as it is in the 7th ſtep, and to ſubtract the rgle* AE × EC *from* AB × BC *it is done, by writing the rgle* AE × EC *after the rgle* AB × BC *from which it is to be ſubtracted, with the negative ſign — minus before it, which expreſſion ſignifies that it muſt be conſidered as taken away from the rgle before it,* (ſee the explanation of the ſign — minus in page 48) *for it muſt be clear to any reaſonable perſon, that a negative in its nature will always deſtroy an affirmative; therefore, to take*

* By both ſides of the equation, is to be underſtood, the quantities on each ſide of the ſign of equality (=) *i. e.* the quantity, or rgle AB × BC is on one ſide, and the quantities, viz. the rgle AE × EC and BE² are on the other ſide of the ſign; ∴ when the ſign of equality (=) is placed between any quantities, the expreſſion produces an equation. See obſervation after the ſign = in page 50.

away

away any quantity already affirmative as in the above instance, is nothing more than to place the negative sign before it as above. The reason why the rgle $AE \times EC$ *is taken away from the 6th step, is this, to leave* BE^2 *alone, as the thesis of the theorem requires, and by so doing the other part of the thesis is produced.*

FINIS.


1 p.2
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2 p.3
A B
3 p.3
B C
D E
4 p.4
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5 p.5
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c a b
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6 p.6
B C
A D
H
E
G F
7 p.6
A
B D
C
8
A B C D
E F G H
p.8
9 p.8
B
C D
A E
10 p.8 9
F B G
A G D
E G
11 p.9 10
A C
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15 p.10 11
k k
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g g
i c h a d
e f
16 p.11 12
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B C
17 p.12
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30 p.19
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31 p.19
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33 p.20
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34 p.20
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37 p.21
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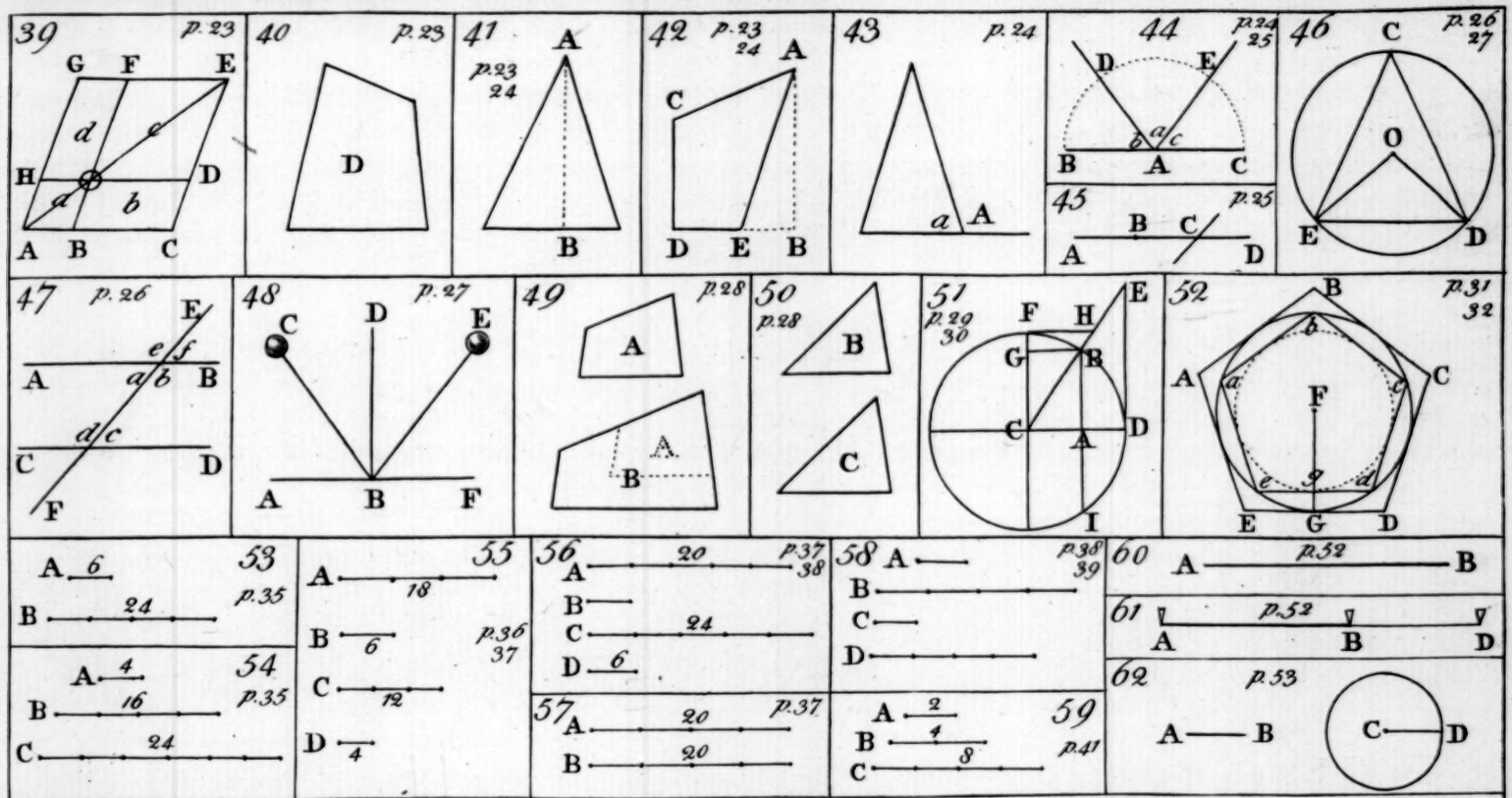
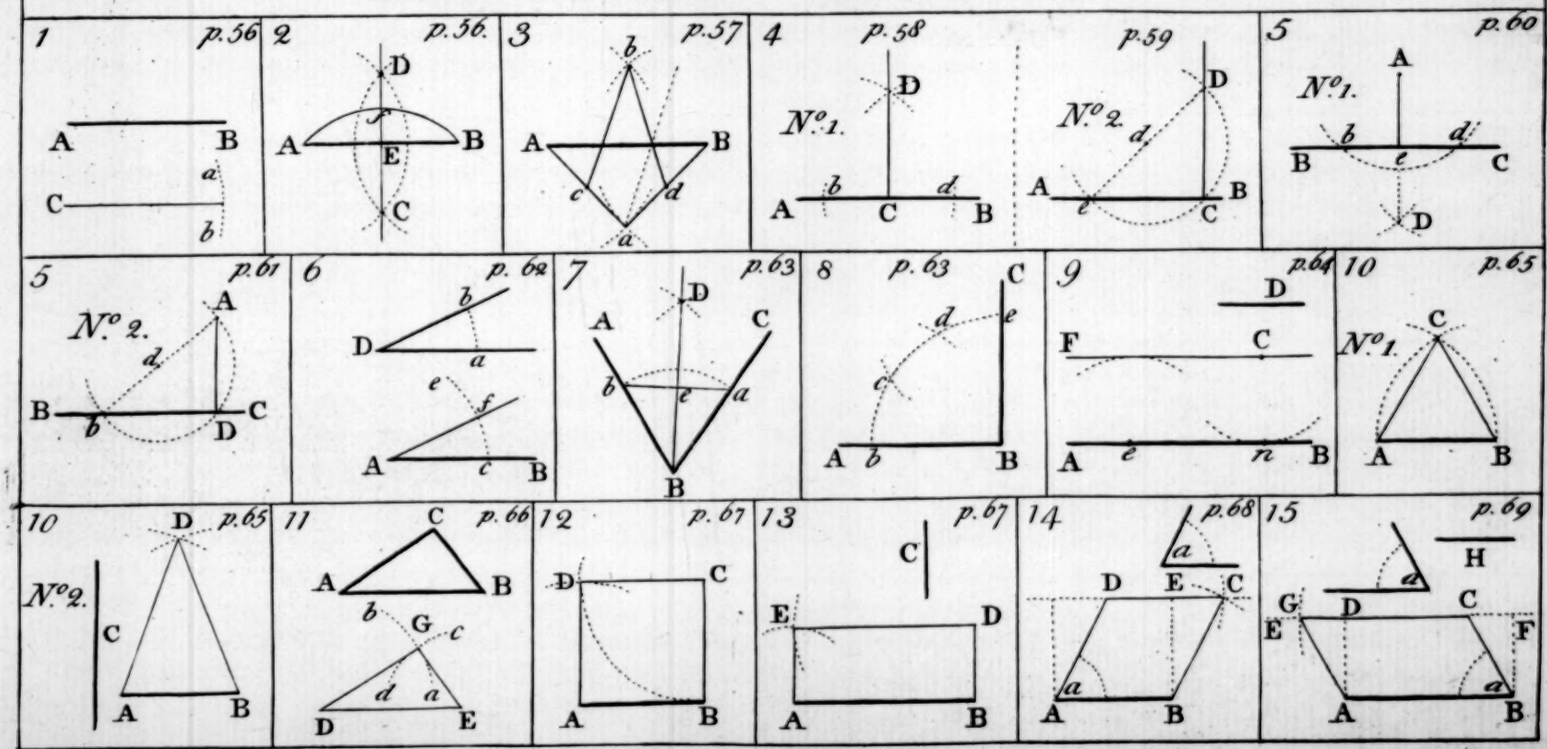
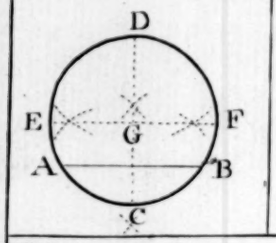
38 p.22
C



PRACTICAL PROBLEMS.



16 p.70



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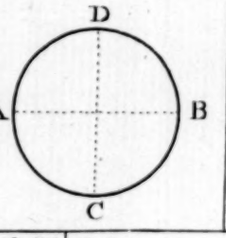

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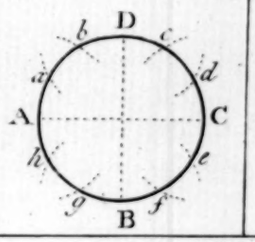

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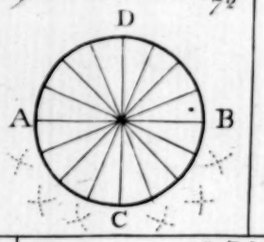

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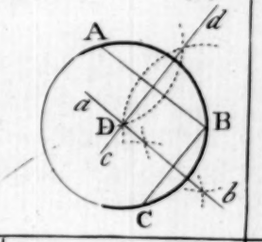

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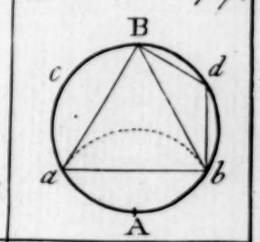

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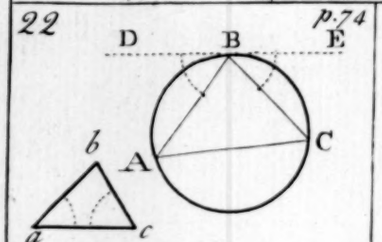

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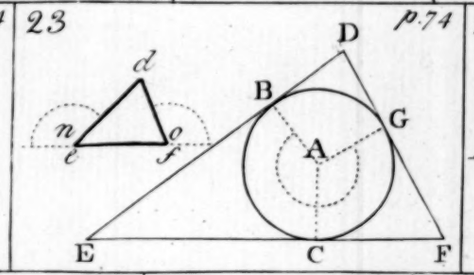

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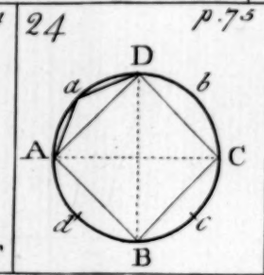

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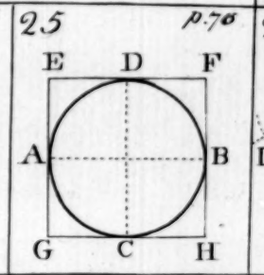

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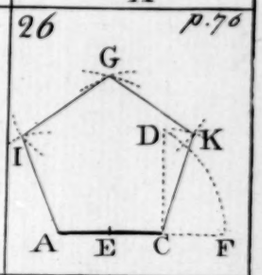

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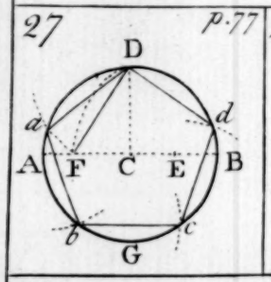

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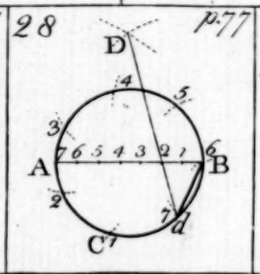

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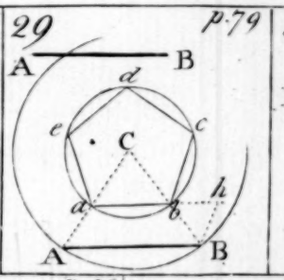

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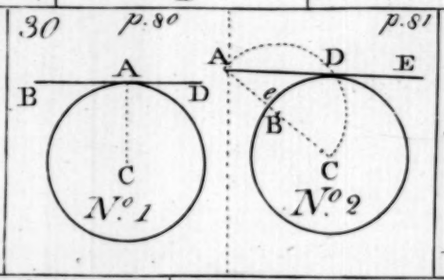

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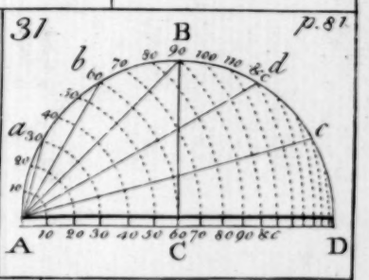

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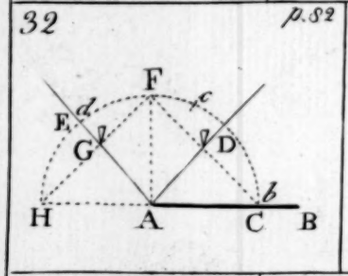

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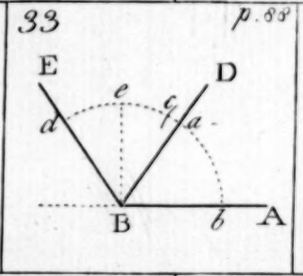

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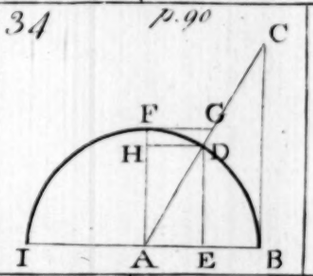

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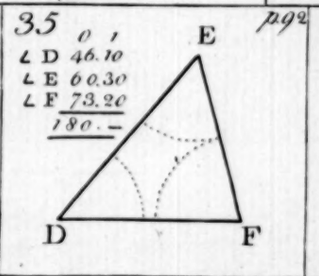

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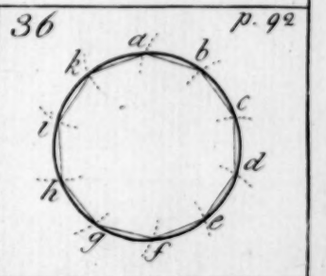

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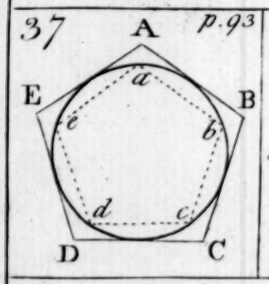

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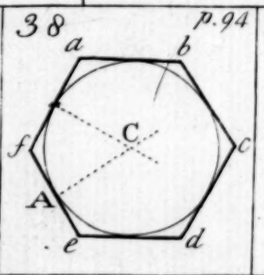

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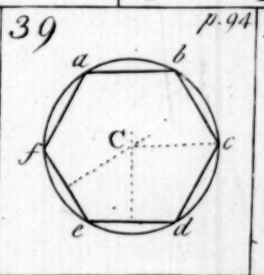

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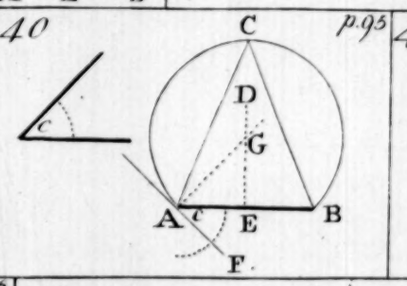

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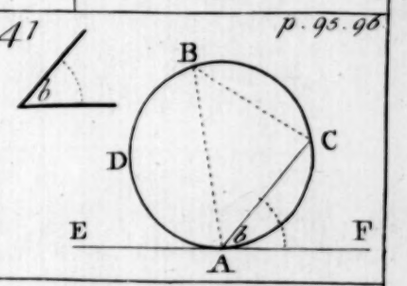

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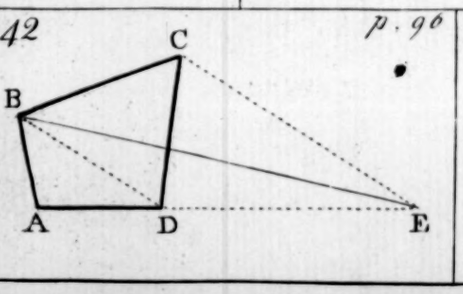

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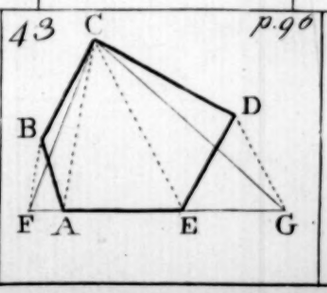

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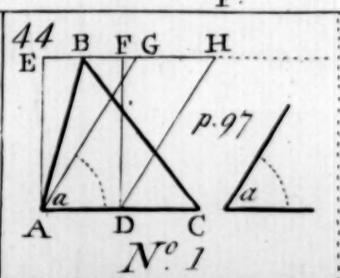

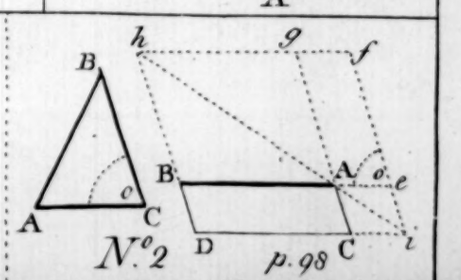

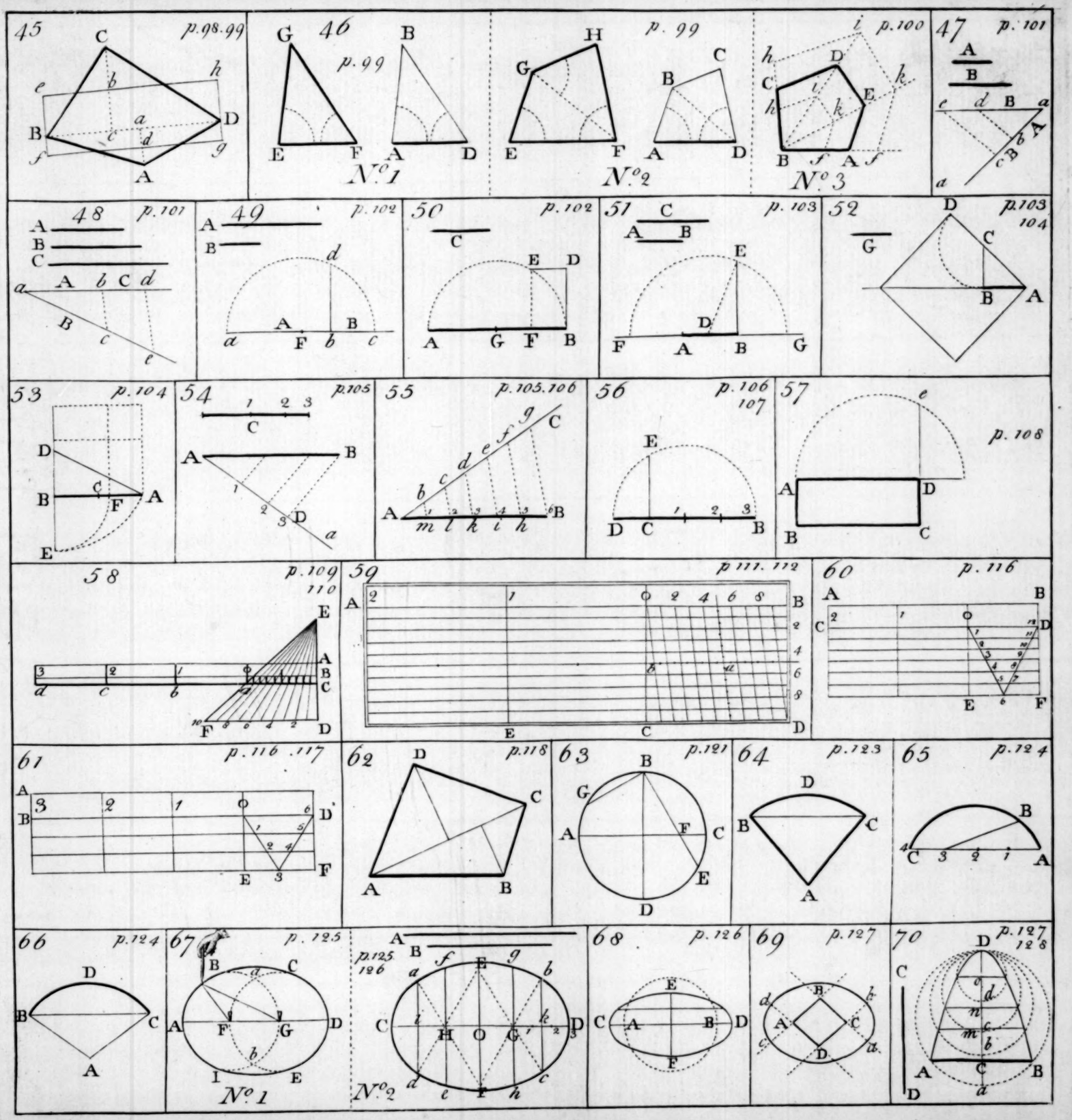
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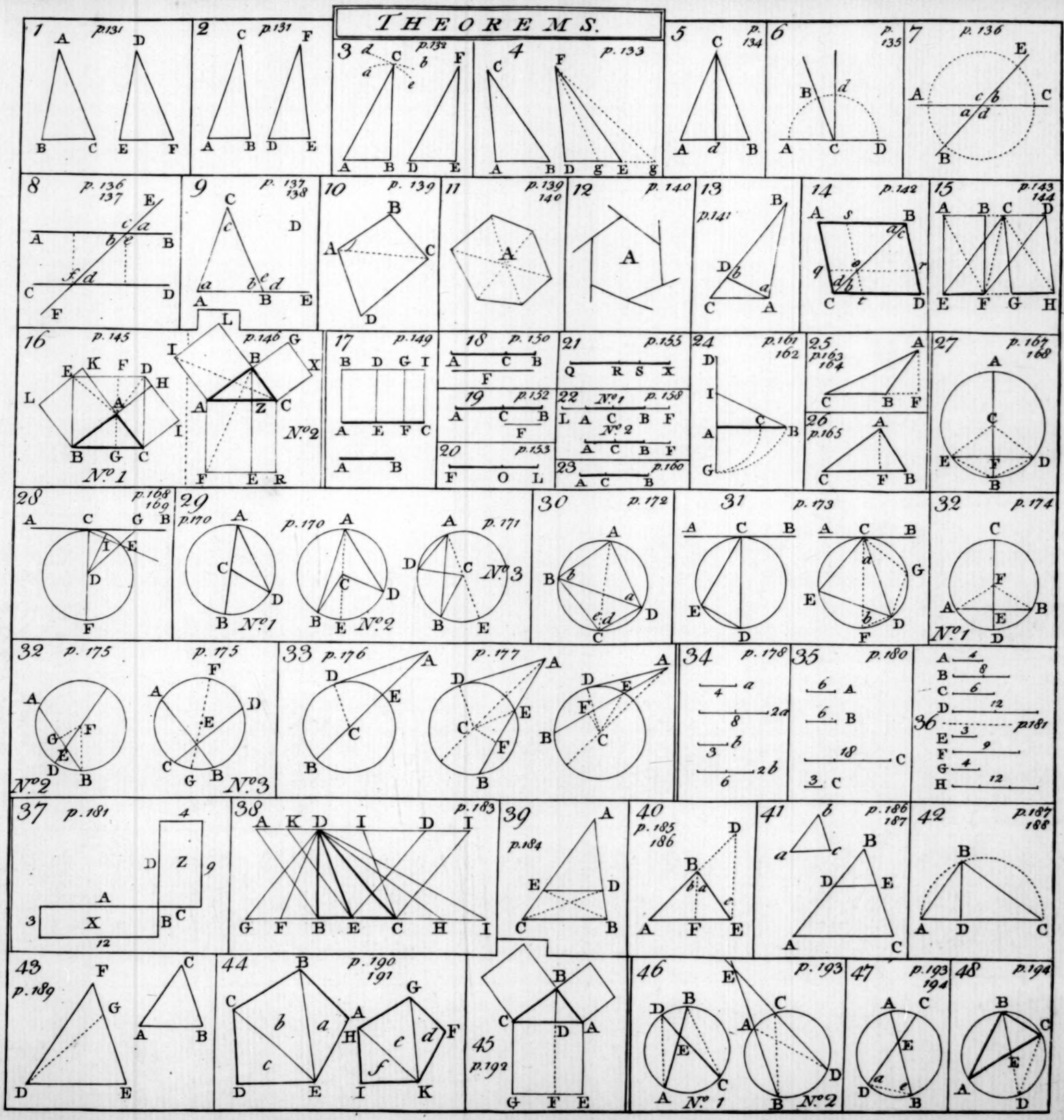
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